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THE PERFORMANCE OF AN AIR-AIR EJECTOR ACCORDING TO A QUASI-ONE-DIMENSIONAL THEORY

by

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Communicated by Prof. T. R. F. Nonweiler

April, 1967

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F.M.3877 FLUID MOTION SUB-COMMITTEE F.M.3877

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SUMMARY

A quasi-one-dimensional theory is developed for the flow in an air-air ejector with sonic injection. It is based on the concept of "general" or "two stream" choking and deals with the constant area compressible mixing of two streamsof the same gas with constant specific heats. Some numerical solutions are presented for the case of an air-air ejector. The effects of the total pressure ratio and total enthalpy ratio between the injected streams are determined for various geometries. The maximum flow or choked solutions are also investigated. The theory is extended to cover the cases of supersonic injection and the mixing of two different gases. A few numerical results for supersonic injection are presented.

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Nomenclature/

Nomenclature

A	cross-sectional area
f	momentum function of $\tilde{w}_{ij}$
H	total enthalpy
h	total enthalpy ratio primary/secondary
h_3	total enthalpy ratio mixed/secondary
K	energy ratio $M^2 (\gamma - 1)/2$
M	Mach number
m	mass flow ratio secondary/primary
P_b	back pressure
P_t	total pressure
p	static pressure
v	velocity
y	mass flow density function of $\tilde{w}_{ij}$
z	area ratio A_{11}/A_3
z'	area ratio A_{12}/A_3
z''	area ratio A_{12}/A_{22}
γ	ratio of specific heats Cp/Cv
π	total pressure ratio P_{t11}/P_{t2}
π'	total pressure ratio P_{t12}/P_{t2}
π''	total pressure ratio P_{t3}/P_{t2}
$\tilde{w}_{ij}$	isentropic pressure ratio p/P_t for the ith stream at the jth plane
ρ	density

Suffix

1	refers to the primary before mixing
2	refers to the secondary before mixing
3	refers to the mixed stream at the cross-sectional plane 3
b	refers to the condition into which the ejector exhausts
ij	refers to the ith stream at the jth plane

1. Introduction

The effects that occur when two gas streams are allowed to mix dynamically in a closed channel have been recognised and made use of for some time. Examples of applications include such devices as ejectors, jet pumps, thrust augmentors, etc. The design and optimisation of these machines requires a theory and a method of analysis which are reasonably accurate, and yet, not so complex that they become impossible to apply. Ideally a theory that would supply an explicit algebraic solution would serve best. To date, no such theory has emerged and it is generally recognised that all solutions have to be obtained either numerically or graphically. When using a numerical method for optimisation it must be possible to make surveys of the effects of the various controlling parameters on the solution. When the analysis is very complex this procedure becomes very tedious if not impossible.

The problem is one of finding a suitable compromise between theories that describe the flow in great detail for a given situation and that are highly complex, and theories that are simple but do not explain all the effects that are found to take place. The approaches made so far fall into two main groups, one on either side of the optimum. Firstly there are those that deal with the actual mechanism of the mixing process. These methods are similar to those used for free jets and often involve a boundary layer type of analysis of the mixed region between the two streams and a two-dimensional description of the streams themselves. This approach is mainly used in low speed work or where compressibility effects are not great. The analysis is extremely complicated and the solution obtained not only corresponds to a unique set of control parameters but is also heavily dependent upon the actual geometry of the situation.

In their simplest form the methods in the second group reduce to satisfying the one-dimensional equations for conservation of mass, momentum and energy between the two streams just prior to mixing and the combined stream when it is entirely mixed. Both streams are assumed to consist of the same perfect gas with constant specific heats. These solutions do not take into account the effects of viscosity or the mechanism of the mixing process. As might be expected, such theories do not give very satisfactory results in predicting experimental flows or in the explanation of the effects observed. The analysis becomes especially simplified in two special cases; mixing at constant pressure and mixing at constant cross-sectional area. In both cases the simplification is due to the wall pressure term in the momentum equation becoming trivial.

The comparative simplicity of the one-dimensional approach makes the second group of methods preferable for use in an overall study of ejector flow processes. Several improvements have been made to the basic theory to try to rectify the short comings mentioned above. Most of these improvements, in dealing with mixing at constant cross-sectional area, divide the total solution into separate regimes. Each regime represents a particular flow pattern and is described by its own theory and its own set of relations. The main distinguishing factor between the regimes is the behaviour of the primary stream on entry to the mixing duct. There is a slight difference in the division of the solution into regimes between the cases of sonic and supersonic injection.

In sonic ejectors, that is those with convergent injection nozzles, four major flow regimes appear. Firstly, if all the pressure differences in the system are small, the flows will be subsonic and both streams will enter the mixing zone at the same static pressure. This is known as the subsonic

regime./

regime. If the pressure against which the ejector exhausts is lowered, it increases the mass flow rates, the primary stream becomes choked at the throat of the injection nozzle and enters the duct in an underexpanded state. When the primary is underexpanded, but the flow is still dependent on the exhaust pressure, the flow is then said to be in the mixed regime. As the exhaust pressure is lowered still further there occurs a limit beyond which it no longer has any effect upon the flow. When this state is reached the ejector is said to be working in the supersonic regime. This is the solution of maximum mass flow for fixed upstream conditions. If, also, the total pressure ratio between the two streams is large or the injection area of the secondary stream is small, the expansion of the primary stream may tend to cut off the secondary stream altogether. Solutions in which this effect is dominant are said to be in the base pressure regime.

Ejectors in which the primary stream enters the mixing duct through a convergent-divergent nozzle are called supersonic ejectors. The naming of the flow regimes that occur in these is due to Fabri. The solution, which is independent of the downstream pressure, is split up into three sub-groups determined by the state of the primary stream on the injection to the mixing zone. Where the primary is underexpanded, the regime retains the name of the supersonic regime. When the static pressures of the two streams are equal on entry and the secondary stream is choked, the flow is in what is termed the saturated condition. If the primary stream is overexpanded the secondary will again be choked and the flow is in the supersaturated supersonic regime. The mixed regime, as before, refers to that part of the solution in which the primary stream is choked and the flow is dependent on the exhaust pressure. As might be expected the mixed regime has three sub-divisions, one corresponding to each of the supersonic regimes above. These are not usually given specific names.

The main failures of the simple one-dimensional approach are in its predictions of the solutions in the supersonic regime and its predictions of the base pressure solutions. The improved theories that try to rectify these faults, all deal with the region of the flow at the upstream end of the mixing duct where the two streams first come into contact. The main differences between the various theories lie in the mechanisms whereby they limit the secondary mass flow.

In dealing with the supersonic solutions, Fabri[1] considered both streams to behave isentropically during the primary expansion. He applied the conditions of conservation of mass and momentum across the expansion which was considered to continue until the secondary stream became choked. This theory seems to have had some reasonable success with certain ejector configurations, but does not prove satisfactory over the whole supersonic regime. It has been found that no one-dimensional theory is effective in predicting the base pressure solutions. This is not surprising since it is here that the viscous effects between the two streams become dominant and no one-dimensional theory takes these into account. To deal with the base pressure solutions, Messrs. Chow and Addi[2] developed a quasi one-dimensional theory which treated the primary expansion by the method of characteristics using a one-dimensional treatment of the secondary stream as a boundary condition. A preliminary inviscid solution is obtained using similar assumptions to Fabri. Then, a boundary layer type of analysis is made of the viscous region along the interstream boundary thus obtained. This is superimposed on the inviscid flow and the whole flow is adjusted accordingly. This approach was found to give good results and gave better predictions of experimental flows than that of Fabri. It suffers, however, from the fact that the calculations involved to predict just a single flow are very complex and take quite some time, even with the use

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of a high speed digital computer. Being a two-dimensional method, it is also heavily dependent on geometry and hence, as a method of surveying the total solution, it is of limited use.

Another approach to the problem of describing the expansion of the primary stream one-dimensionally was introduced by Pearson, Holliday and Smith[3] in a paper on a theory for ejector nozzles with short shrouds. Here the primary stream is considered to expand polytropically until the exit of the short shroud, where it emerges at the same static pressure as the secondary. The secondary stream is treated isentropically and the effects of viscosity and mixing between the two streams are ignored. In the supersonic regime the mass flow in the secondary stream is limited by an interstream choking effect. This takes place in the exit plane of the shroud or mixing duct while the secondary is still subsonic.

It is the intention of this paper to put forward and examine a theory and method of analysis which will explain with reasonable accuracy the effects that occur when two streams of the same perfect gas mix dynamically at constant cross-sectional area. The streams are considered to have different total enthalpies as well as different total pressures. The theory is of a similar nature to that of Pearson, except that it deals with ejectors with long shrouds or mixing chambers where the flows have time to become completely mixed. The limits within which the solutions are applicable are sketched and discussed. Using the resulting method of analysis, a numerical survey of the problem is made for sonic and subsonic injection over a wide range of conditions. It is of academic interest to note that there exist in the mixed regime some supersonic alternative solutions for which there is an overall gain in total pressure without infringing on the second law of thermodynamics. The question of the physical existence of such solutions is also discussed.

2. Theory

2.1 The theoretical system and assumptions

The theory is developed for the case of sonic injection, but, as it will be shown later, it may be easily extended to the supersonic case. The basic approach to the problem was to take a theoretical system, build up a mathematical model for it and then find a method of analysis that would allow an extensive survey of its performance. The system that was chosen is shown in Fig. 1. It consists of two reservoirs of a gas exhausting via a common channel into a third reservoir or the atmosphere. The mixing process between the two streams takes place in a duct of constant cross-sectional area. The two exhausting reservoirs are connected to this duct by two convergent nozzles whose combined throat area is equal to the cross-sectional area of the duct. Hence using the notation of Fig. 1.

$$A_1 + A_2 = A_3$$

For a one-dimensional treatment the geometry of the flow may thus be expressed in a single parameter such as α $(= A_1/A_3)$.

For this system the controllable parameters may be taken as: the geometry, the total pressures and the total enthalpies of the gases in the two reservoirs, and the back pressure into which the mixing tube exhausts. The analysis for the problem in the general state is very complex, and in order to develop a mathematical model with a simple analysis the following assumptions are made: the entire process is treated one-dimensionally. The same perfect gas with constant specific heats is used throughout. The boundary layer and frictional effects at the walls are assumed to be negligible. The length of the mixing tube is assumed to be such that the flow is completely mixed at the exit.

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These assumptions reduce the problem to one of four basic parameters: π, the total pressure ratio and h, the total enthalpy ratio between the two exhausting reservoirs, z the geometric parameter, and p_b, the back pressure. The reservoir with the higher total pressure and the stream issuing from it are designated the primary, and the other reservoir and stream the secondary. In the range of conditions investigated, the primary total enthalpy is always greater than that of the secondary.

For the purposes of analysis the flow is divided into three sections as shown in Fig. 2b. The first section covers the flows from the reservoirs to the plane of entry to the mixing tube. The second covers the region in which the mean pressures of the two streams are not equal, and the third from the plane of equal pressures until the mixing is complete.

In the first region the streams are assumed to flow out of the reservoirs isentropically and arrive at the entrance to the mixing tube having suffered no losses. These flows may be considered to be dependent upon the static pressures at the plane of entry.

If both streams enter the mixing duct subsonically it is assumed that the static pressures of the two streams are equal at the plane of entry. In such cases the secondary region mentioned above vanishes and the flow goes straight onto the mixing process. If, however, the primary stream becomes choked at the plane of entry, the static pressures at this plane need not be equal. There would then follow a process in which the two mean pressures would be equalised. This is the second region depicted in Fig. 2b, and it ends at the plane where the static pressure of the combined stream first becomes uniform. The process consists of a rapid expansion of the primary stream, during which the mean pressures of both streams fall, though that of the secondary only slightly. Due to the rapidity of the process it is assumed that there is no mass or energy transfer between the two streams. Momentum is transferred but only by the action of the pressure forces on the boundary between the two streams. Since the mixing duct is parallel sided this momentum transfer must be equal and opposite. The secondary stream remains subsonic during this process and is assumed to be described by the one-dimensional isentropic area-velocity relation. The primary expansion on the other hand is assumed to be irreversible and the solution for the state of the stream at the end of the expansion may be determined by simultaneously satisfying the following conditions:-

that the mean static pressures of the two streams are equal at the end of the process,

that the cross-sectional area of the mixing duct is constant,

that the momentum transfer during the process is equal and opposite,

that both mass and energy are conserved within the stream.

After the two mean pressures have become equal there follows a region of mixing at constant area by the end of which the static pressure across the cross-section is assumed to have become uniform. This is the third region of Fig. 2, and it is analysed by simply equating the influx of mass, momentum and energy to the respective effluxes within the combined stream when it is entirely mixed. The analysis and the relevant equations for each section will now be developed.

2.2/

2.2. Theory

Consider the system as shown in Fig. 2. The parameters which must be given definite values in order to specify a unique condition of flow are: the total pressures and enthalpies of the exhausting reservoirs, the ratio of the throat areas of the two injection nozzles and the back pressure into which the mixing tube exhausts. Making use of these the problem may be uniquely specified in terms of four non-dimensional parameters:-

Primary total pressure/secondary total pressure,	π
Primary total enthalpy/secondary total enthalpy,	h
Primary injection area/area of mixing tube,	z
The back pressure/secondary total pressure,	P_b/P_{t2}

These four parameters will hereafter be referred to as the control parameters. Their theoretical possible ranges and the ranges within which their effects are investigated are given in the table below.

Parameter	Theoretical range	Range investigated
π	1 to infinity	1.1 to 6.0
z	0 to 1.0	0.01 to 0.9
h	0 to infinity	1.0 to 5.0

Given the values of these parameters the equations describing the flow should become soluble. These equations will now be derived.

In the first region the two streams flow isentropically from their respective reservoirs to the entrance of the mixing tube. These flows are governed by the static pressure at this plane, which, it was mentioned above, may or may not be equal. Using the notation defined above and referring to Fig. 2, the equation for the case of equal pressures may be written,

$$p_{11} = p_{21}.$$

or

$$\tilde{w}_{11} P_{t11} = \tilde{w}_{11} P_{t21}$$

where $\tilde{w}_{ij} = p_{ij}/P_{tij}$ for the ith stream and jth plane.

This may be written,

$$\pi . \tilde{w}_{11} = \tilde{w}_{21} . \qquad ...(1)$$

In the cases where the pressures are unequal, the primary stream is choked and its static pressure at plane 1 is assumed to be critical. Hence

$$\tilde{w}_{11} = (2/(\gamma + 1))^{\gamma/(\gamma - 1)} \qquad ...(1a)$$

where γ is the ratio of the specific heats Cp/Cv.

Since/

Since the flows are isentropic in this region their equations of motion maybe written,

$$P_{t10} = P_{t11} \qquad \ldots(2)$$

and

$$P_{t20} = P_{t21} . \qquad \ldots(3)$$

The flows are thus described either by Eqns. (1), (2) and (3) or by Eqns. (1a), (2) and (3). The problem is at this stage indeterminate, for the solution is also dependent upon the conditions downstream, and it is these conditions that will determine the value of the pressure at the entrance to the mixing tube, that is $\tilde{w}_{21}$. Since it has not been found possible to obtain an expression for $\tilde{w}_{21}$ in terms of the downstream conditions, values are to be assumed for $\tilde{w}_{21}$ and the downstream conditions to be determined for these values.

In cases where the primary stream is choked at plane 1 and the static pressures are not equal, an expression which relates the states of the two streams at plane 2 may be derived. Consider the flow in region 2. The secondary stream is subsonic at plane 1 and remains so during the expansion. Hence it will be considered to behave isentropically and obey the well known area-pressure relation,

$$\frac{A}{A^*} = \frac{(1 - \tilde{w}^{(\gamma - 1)/\gamma})^{\frac{1}{2}} . \tilde{w}^{1/\gamma}}{((\gamma - 1)/2)^{\frac{1}{2}} (2/(\gamma + 1))^{\gamma + 1}} .$$

Using this relation and introducing the mass flow density function y_{ij} in place of the expression

$$(1 - \tilde{w}_{ij}{}^{(\gamma - 1)/\gamma})^{\frac{1}{2}} . w_{ij}{}^{1/\gamma}$$

the equation describing the isentropic contraction of the secondary stream becomes

$$\frac{1 - z'}{1 - z} = \frac{y_{21}}{y_{22}} . \qquad \ldots(4)$$

For the irreversible expansion of the primary, continuity states that

$$A_{11}\, \rho_{11}\, v_{11} = A_{12}\, \rho_{12}\, v_{12} \qquad \ldots(5)$$

which may be combined with the energy equation

H_{11}/

$$H_{11} = H_{12} \qquad \ldots(6)$$

and rewritten in the form

$$z\, P_{t11}\, y_{11} = z'\, P_{t12}\, y_{12}\,. \qquad \ldots(6a)$$

For the static pressures to be equal at plane 2,

$$p_{12} = p_{22} \qquad \ldots(7)$$

or

$$P_{t12}\, \tilde{w}_{12} = P_{t22}\, \tilde{w}_{22}\,. \qquad \ldots(7a)$$

Since the two streams are contained in a parallel sided duct, the momentum exchange between them must be equal and opposite, and hence the momentum fluxes across planes 1 and 2 must also be equal.

$$A_{11}\left(p_{11}+\rho_{11}v_{11}^2\right) + A_{21}\left(p_{21}+\rho_{21}v_{21}^2\right) = A_{12}\left(p_{12}+\rho_{12}v_{12}^2\right) + A_{22}\left(p_{22}+\rho_{22}v_{22}^2\right)\,. \qquad \ldots(8)$$

Using the substitution $f_{ij} = \tilde{w}_{ij}\,(1 + 2\gamma\,(\tilde{w}_{ij}^{\,(1-\gamma)/\gamma} - 1)/(\gamma - 1))$ for the momentum flow function, and the others mentioned above, this equation may be written,

$$z\, P_{t11}\, f_{11} + (1 - z)\, P_{t21}\, f_{21} = z'\, P_{t12}\, f_{12} + (1 - z')\, P_{t22}\, f_{22}\,. \qquad \ldots(8a)$$

Since the secondary is isentropic $P_{t21} = P_{t22} = P_{t2}$ say. Further, from Eqn. (6),

$$P_{t12} = \frac{z y_{11}}{z' y_{12}}\, P_{t11}\,. \qquad \ldots(6b)$$

Substituting this into Eqn. (8a),

$$z\,\pi\, f_{11} + (1 - z)\, f_{21} = z\left(y_{11}/y_{12}\right)\pi\, f_{12} + (1 - z')\, f_{22} \qquad \ldots(8b)$$

and into Eqn. (7a),

$$\pi\, z\, y_{11}\, \frac{\tilde{w}_{12}}{z'\, y_{12}} = \tilde{w}_{22}\,. \qquad \ldots(7b)$$

Now/

Now from Eqn. (4),

$$z' = 1 - (1 - z)\, y_{21}/y_{22} . \qquad \ldots(4a)$$

Substituting into Eqn. (8b) for z',

$$\frac{z\,\pi}{1 - z} f_{11} + f_{21} = \frac{z\,\pi}{1 - z} y_{11} \frac{f_{12}}{y_{12}} + y_{21} \frac{f_{22}}{y_{22}} \qquad \ldots(8c)$$

and into (7b),

$$z\,\pi\, y_{11}\, \tilde{w}_{12} = y_{12}\, \tilde{w}_{22} \left(1 - (1 - z) y_{21}/y_{22} \right). \qquad \ldots(7c)$$

Eqns. (7c) and (8c) are two equations relating the variables $\tilde{w}_{21}$, $\tilde{w}_{12}$ and $\tilde{w}_{22}$ or functions thereof. Eliminating $\tilde{w}_{21}$ there emerges a relation connecting $\tilde{w}_{12}$ and $\tilde{w}_{22}$, in fact connecting the states of the two streams at the plane of equal pressures.

In both cases, when the primary is choked and when it is not, from consideration of the upstream conditions alone the solution is indeterminate at plane 2. For a given set of stagnation conditions and a given geometry there will then exist a definite relation between the states of the two streams at plane 2. There will thus be a series of solutions corresponding to different downstream conditions. So by assuming values for $\tilde{w}_{21}$ at the start of the actual mixing it should be possible to find the corresponding values of back pressure at the end of mixing. For a fixed value of $\tilde{w}_{21}$ the state of the secondary stream in region 1 is fixed and the state of the primary stream at the equivalent value of $\tilde{w}_{11}$ may be determined. If this state is found to be subsonic it is adopted and both streams enter the mixing tube subsonically and at the same static pressure. Thus the expansion of region 2 does not exist and planes 1 and 2 of Fig. 2b are coincident giving the situation depicted in Fig. 2a. If, on the other hand, it is found that the primary stream would be supersonic at the equivalent value of $\tilde{w}_{11}$, it is assumed that it has become choked and is in the sonic state at plane 1. Thereafter the static pressures are equalised by the expansion process of region 2. Having assumed a value for $\tilde{w}_{21}$ the values of $\tilde{w}_{12}$ and $\tilde{w}_{22}$ may be found from Eqns. (7c) and (8c). Hence the states of the two streams at plane 2. The degree of irreversibility in the primary expansion may be determined by finding the loss in total pressure, which in turn may be found from Eqn. (6b).

$$\frac{P_{t11} - P_{t12}}{P_{t11}} = 1 - \frac{z\, y_{11}}{z' y_{12}} .$$

After/

After plane 2 the streams are allowed to mix. The static pressure is assumed to be uniform across both the plane at the end of the mixing tube, plane 3, and the plane at the start of the mixing process, plane 2. Since the mixing takes place in a parallel sided duct, all that is necessary in a one-dimensional treatment is to satisfy the conditions of conservation of mass, momentum and energy between these planes assuming that they are far enough apart for the properties of the mixed stream to have become uniform across the cross-section. These conditions give rise to the following equations:-

Continuity,

$$A_{12}\,\rho_{12}\,v_{12} + A_{22}\,\rho_{22}\,v_{22} = A_3\,\rho_3\,v_3 \qquad \ldots(9)$$

Energy,

$$A_{12}\,\rho_{12}\,v_{12}\,H_{12} + A_{22}\,\rho_{22}\,v_{22}\,H_{22} = A_3\,\rho_3\,v_3\,H_3 \qquad \ldots(10)$$

Momentum,

$$A_{12}\left(p_{12} + \rho_{12}\,v_{12}^2\right) + A_{22}\left(p_{22} + \rho_{22}\,v_{22}^2\right) = A_3\left(p_e + \rho_3\,v_3^2\right) \qquad \ldots(11)$$

Using the substitutions for $\tilde{w}_{ij}$, y_{ij} and f_{ij} given above together with the following substitutions

$$h_3 = H_3/H_{21},\ \pi' = P_{t21}/P_{t2},\ \pi'' = P_{t3}/P_{t2} \text{ and } z'' = A_{12}/A_{22}$$

Eqns. (9), (10) and (11) may be written:-

$$y_{22} + z''\,\pi'\,y_{12}\,h^{-\frac{1}{2}} = (1 + z'')\,\pi''\,y_{33}\,h_3^{-\frac{1}{2}} \qquad \ldots(9a)$$

$$y_{22} + z''\,\pi'\,y_{12}\,h^{\frac{1}{2}} = (1 + z'')\,\pi''\,y_{33}\,h_3^{\frac{1}{2}} \qquad \ldots(10a)$$

$$f_{22} + z''\,\pi'\,f_{12} = (1 + z'')\,\pi''\,f_{33}\,. \qquad \ldots(11a)$$

Given that all the quantities on the left-hand sides of these equations are known, there are three equations and three unknowns. The unknowns are π'', h_3 and $\tilde{w}_{33}$, the quantities y_{33} and f_{33} being simple functions of $\tilde{w}_{33}$. Eliminating π'' and h_3 between the Eqns. (9a), (10a) and (11a), there emerges a relation which gives a quadratic solution for the state of the mixed stream. In general one root yields a supersonic solution and the other a subsonic solution. Once values for $\tilde{w}_{33}$ have been found the other properties of the mixed stream may be determined by substitution back into Eqns. (9a) to (11a).

$$(y_{33}/f_{33})^2 = \frac{y_{22}^2 + z''\,\pi'\,(h^{\frac{1}{2}} + h^{-\frac{1}{2}})\,y_{22}\,y_{12} + z''^2\,\pi'^2\,y_{12}^2}{(f_{22} + z''\,\pi'\,f_{12})^2}\,. \qquad \ldots(12)$$

Considering/

Considering the flow as a whole we now have a set of equations that will, for given geometry and stagnation conditions, supply a series of quadratic solutions for the state of the mixed flow corresponding to a set of assumed values of the secondary injection pressure p_{21}.

2.3 Method of analysis

Due to the complexity of the above system of equations it was not found possible to obtain an explicit algebraic solution to them. Therefore a numerical method was used to make a survey of the effects of the control variables on the solution. This method, developed for use with a high speed digital computer, will now be described.

For a given set of control variables it was shown above that it is possible to find the states of the mixed stream corresponding to assumed values of the secondary injection pressure. To obtain this series of solutions the following process is carried out.

A high value for the secondary injection pressure is selected, usually just below the secondary total pressure. This gives a small mass flow in the secondary stream. It is then checked whether or not the primary stream is choked at this pressure. If it is, the procedure outlined in the next paragraph is carried out straight away. If, on the other hand, it is not, the following procedure is adopted. The two injection pressures are assumed to be equal and the chosen pressure p_{21}, is used to define the states of both streams at plane 1. This is the subsonic regime depicted in Fig. 2a, where both streams enter the mixing tube subsonically. The quadratic for the state of the mixed stream is then solved yielding two solutions, a subsonic one and another one which is, in general, supersonic. In the general solution the Mach number of the supersonic solution increases as the value of the secondary injection pressure is raised. In certain cases the supersonic solution possesses a singular point over which the Mach number passes from plus infinity to minus infinity as p_{21} is increased. The supersonic solution is assumed not to exist for solutions with values of secondary injection pressure above that at which the singularity occurs. For those solutions that do exist, the following properties of the mixed flow were computed;

the Mach number,
the total pressure,
the total enthalpy,
the mass flow ratio, secondary/primary,
the static pressure,
and the critical area ratio.

The above process is repeated for decreasing values of p_{21}, the secondary injection pressure, until the primary stream becomes choked in plane 1. At this stage the static pressures of the two streams are no longer equal on injection and the primary stream is made to expand as shown in Fig. 2b. This process is described by Eqns. (7c) and (8c) above. Using the current value of p_{21}, and hence of $\tilde{w}_{21}$, these equations are solved by an iterative process to find the states of the two streams at the end of the expansion. This consists of choosing successive values for $\tilde{w}_{22}$ in such a way that the two values of $\tilde{w}_{12}$, found from Eqns. (7c) and (8c), tend to converge and coincide. When they are sufficiently close to coinciding the process is stopped and the current values of $\tilde{w}_{22}$ and $\tilde{w}_{12}$ used to define the states of the streams at plane 2. The mixing quadratic, Eqn. (12), is then solved as before. The process is then repeated for

increasingly/

increasingly smaller values of the secondary injection pressure p_{21}. Because the primary stream is choked at plane 1, its mass flow is now fixed and the reduction of p_{21} only affects the secondary mass flow. As the chosen values of p_{21} fall the two solutions of the mixing quadratic gradually converge towards the sonic condition. Within the range of solutions investigated, it is found that there are two mechanisms that prevent the secondary injection pressure falling indefinitely and that limit the mass flow that can be passed through the system. Firstly, the two roots of the quadratic may coincide or become imaginary, in which case it is assumed that the mixed flow has become choked at the exit of the mixing duct. Alternatively, the two values of $\tilde{w}_{12}$ in the iterative procedure for the primary expansion may be found to converge and then diverge, without ever becoming equal. In such cases the two streams could never reach a state of equal pressures and it is assumed that such a situation would be prevented from taking place by the two stream choking effect occurring when the two values of $\tilde{w}_{12}$ are only just able to coincide. These phenomena are discussed further below. When either of these limiting effects occur, the process of taking successively lower values of p_{21} is stopped, and the solution for that particular set of control variables considered complete.

3. Solutions Obtained Using the Theory Presented

3.1 A full examination of a few typical solutions

To illustrate the forms that the general solution may take in the various flow regimes, some specific examples exhibiting each of the choking effects will be discussed. The solutions chosen for this purpose are those corresponding to the following set of control variables:-

Injection nozzle area ratio A_1/A_3 or	z	=	0.5
Total pressure ratio	π	=	1.7
Total enthalpy ratio	h	=	1.0, 1.5, 2.0, 3.0. 4.0.

The solutions are presented in terms of the static pressure after mixing, p_3, hereafter called the exhaust pressure and the secondary injection pressure, p_{21}. For the purposes of plotting these solutions in a non-dimensional form all the pressures are normalised with respect to the secondary total pressure. As may be seen on inspecting Fig. 6, each solution corresponding to a fixed set of parameters is double valued with respect to the injection pressure, p_{21}. The upper branches are subsonic solutions and the lower ones are supersonic solutions. The minima in p_{21} represent the choked solutions and they are also those of maximum mass flow. The overall mass flow is a single valued function of p_{21} and increases with decreasing values of that pressure. This may be seen on looking at the curve for m, the mass flow ratio, secondary over primary, versus p_{21} in Fig. 7. Also shown in Fig. 7 are the curves for the total pressure ratio P_{t3}/P_{t1}, the energy ratio after mixing, $K\left(=\frac{\gamma-1}{2}M^2\right)$, and the critical area ratio, A_3^*/A_3, for the case when the total enthalpy ratio, h, is 2.0.

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Consider the flows given by the solutions shown in Fig. 6. To illustrate the various flow regimes we will now consider the effect of the back pressure on the systems represented. When the back pressure is high it is matched by the exhaust pressure and the mixed flow is well subsonic. Both flows are also injected into the mixing tube at subsonic speeds. This is the subsonic regime and all such solutions are dependent on the downstream conditions. Referring to Fig. 6, and selecting a typical value for p_3, of, say, 1.15 P_{t2}, it will be seen that, at this stage, all the solutions shown are in the subsonic regime. The boundary between the subsonic and mixed regimes takes the form of a line, p_{21} = constant, since the pressure at which the primary stream chokes is dependent only on the total pressure ratio π. In the case under consideration, the relevant value of p_{21} is 0.898 P_{t2}. As the back pressure is lowered, and the exhaust pressure with it, all the solutions enter the mixed regime, those of lower total enthalpy leading. While the solutions are in the subsonic regime both the primary and the secondary mass flows increase as p_{21} and p_3 decreases; but in the mixed regime only the secondary mass flow is affected. The solutions in which $p_3 = P_{t2}$ represent the special cases for which the secondary reservoir and the exhaust reservoir may be one and the same. In the mixed regime the primary enters the mixing duct in an underexpanded state. As the exhaust pressure, p_3, is lowered and the solution points move to the left down the curves in Fig. 6, the pressure difference between the streams on injection gets larger, and the solution for the mixed stream tends towards the sonic state.

Considering in particular the solution for $h = 3.0$ and 4.0 in Fig.6, as the exhaust pressure falls below a value of about 0.7 P_{t2} the mixed stream becomes sonic and the system chokes at the exit of the mixing tube. It is thought that, by itself, further lowering of the back pressure will have no effect on the solutions and that the exhaust pressure will remain at the sonic value. Solutions of this type are said to be in the supersonic regime. It should be noted that these choked values are also those of maximum total mass flow and minimum mixed stream total pressure.

As the back pressure is lowered in the solutions for which $h = 1.0$, 1.5 and 2.0, the flows experience the two stream choking effect before the mixed flow becomes sonic. On the graphs this effect is shown by the truncation of the parabolic type curves before their natural minima are reached. The limiting flows in these cases are maximum mass flows with respect to the primary expansion as opposed to the mixing process. The two stream choking effect takes place at the plane of equal static pressures at the end of the primary expansion. It has been shown (see Ref. 3) that the plane wave velocity is stationary in both streams at this plane for the condition of maximum mass flow. When the back pressure falls below the point where this choking takes place, it only effects the flow downstream of the choked plane. The solution for the mixed stream in this case is double valued; one solution being subsonic and the other supersonic. Now, mathematically, the subsonic solution is equivalent to the supersonic one with a normal shock at the end of the mixing duct. This suggests that, for the values of the back pressure less than the subsonic limit of the exhaust pressure, p_3, the supersonic choked solution is adopted together with an oblique shock system at the duct exit to match p_3 and P_b. As the back pressure falls the shock system would get weaker and would vanish as the supersonic value of p_3 is approached. Beyond this point the flow would certainly not be affected and could truly be said to be in the supersonic regime.

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As may be seen from the results presented, all the subsonic solutions and the supersonic ones associated with the two stream choking effect result in mixed stream total pressures in between those of the primary and secondary streams. They also obey the second law of thermodynamics in that they produce enthropy.

3.2 Existence of the supersonic solutions

Now it has been shown that, for each subsonic solution, there exists, mathematically if not in reality, an alternative solution of identical mass flow. The flows corresponding to both solutions differ only in the action of the mixing process and thus in the state of the mixed stream. In most cases the alternative solution is a supersonic one, but in some cases with high values of the secondary injection pressure and low values of the ratio, z, the alternative solutions pass through a singular point where the Mach number of the mixed flow becomes infinite. Points on the other side of this singularity, that is with even larger values of p_{21}, produce alternative solutions with negative Mach numbers. These solutions are assumed to have no meaning.

Looking at the supersonic solutions, theoretically, as the exhaust pressure, p_3, falls from its value at the choked condition, the solutions for the mixed flow tend to become more supersonic and the corresponding values of the secondary injection pressure start to rise This indicates that both the total mass flow and the pressure difference between the two streams on injection will fall. On looking at Fig. 6 it will be seen that the solutions will pass out of the mixed regime into the subsonic regime if p_3 is allowed to fall far enough. Two other special points occur in the general solutions as the exhaust pressure falls. Firstly there comes a point beyond which the total pressure of the mixed stream becomes larger than that of either of the component streams. This point we shall call the point of total pressure gain. The other point is one beyond which the solution no longer obeys the second law of thermodynamics in that it starts to lose entropy. This will be known as the isentropic point. The total pressure gain points and the isentropic points are marked by triangles and circles respectively on the graphs presented in this paper. The order in which these points and the return of the solution into the subsonic regime occur varies from solution to solution. This is well illustrated in Fig. 6.

"Can any of these supersonic solutions be made to occur physically?" is the question that must now be asked. In dealing with the subsonic solution it was natural to assume that they would respond in such a way as to match any changes in the back pressure. But, as in the case of the ordinary convergent nozzle, it would also seem natural to assume that further lowering of the back pressure after the system had already choked would not have any effect, and that the system would remain in the condition of maximum mass flow even when exhausting into vacuum. This would almost certainly be so for those solutions that choke at the end of the mixing process. The case is not quite so clear for those solutions that choke due to the two stream choking effect, but it is assumed that when the back pressure falls below the value of p_3, corresponding to the supersonic version of the choked solution, it will no longer influence the solution.

The conclusion drawn from the above discussion is that the system in the simple form of a constant area mixing duct exhausting straight into a low back pressure or vacuum will never produce a solution in which the state of the mixed flow is more supersonic than that of the maximum flow solution. This

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does not, however, exclude the possibility of obtaining such a solution by making some modification to the system. It might be an interesting experiment to try restricting the total mass flow by narrowing down the exit of the mixing chamber when the ejector is in the choked condition and exhausting into a very low back pressure. The system will then choke at the throat of this variable exit nozzle as shown in Fig. 3. On looking at Fig. 7 where a solution is plotted in terms of the critical area ratio A_3^*/A_3, after mixing it will be seen that for a fixed reduction in exit area, that is a fixed value of A_3^*/A_3, the two alternative solutions have differing values of p_{21}. Thus, they have different rates of mass flow and it will also be noted that it is always the supersonic solution that has the greater mass flow.

However, it is not thought that this process is likely to produce a supersonic solution, for not only would it involve supersonic diffusion but it also requires the existence of a steady supersonic flow not preceded by a choked flow. To obtain supersonic flow in a mixing region it would be necessary to choke the flow upstream of that section. In none of the non-maximum flow solutions does any mechanism occur whereby such choking could take place.

3.3 General survey of the solution for sonic injection

The numerical investigation of the effects of the control variables on the solution is carried out for the mixing of two streams of the same prefect gas. The specific heats of this gas are taken to be constant and their ratio to be 1.4, the same as that for air. One of the main objects of the investigation is to determine the effects of varying each of the control parameters on the way in which the flow chokes. The survey is split into two principal parts. The first consists of obtaining complete solutions for flows with fixed upstream stagnation conditions and fixed geometry. In these solutions the effect of the back pressure is included. The second part is a survey of the maximum flow solutions; that is those in the supersonic regime, where the back pressure has no effect.

3.3.1 The survey of full solutions

In the survey of full solutions, solutions are evaluated over a four-dimensional field. The total pressure ratio, the total enthalpy ratio and the geometric ratio form three of these dimensions. The full solutions are obtained keeping these three fixed and allowing the fourth dimension, the back pressure, to vary. The ranges within which the various parameters are allowed to vary are as follows:-

z,	the geometric ratio,	from 0.2 to 0.8
π,	the total pressure ratio,	from 1.7 to 2.0
h,	the total enthalpy ratio,	from 1.0 to 4.0

As explained above, it is not possible to select values for the back pressure directly, so values of the secondary injection pressure, p_{21}, are selected instead. The procedure of obtaining a full solution is started by taking a value of p_{21} just below the secondary total pressure. The problem is then solved for this and successively lower values of p_{21} until the supersonic regime is reached.

The ranges of z and h over which solutions are obtained are reasonably comprehensive. But, for the total pressure ratio, π, a fairly narrow band was chosen, so that it covered the critical value, 1.89 in the case where $\gamma = 1.4$.

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The results were plotted on the $p_3 - p_{21}$ field. The choice of p_3 was made because it gave a good illustration of the possible effects of the fourth control variable, the back pressure, and that of p_{21} because it gave a clear view of the double valued nature of the solution. On each solution the points at which the following effects occurred were marked:-

the choking of the primary stream at the entrance to the mixing tube,

the choking of the whole flow due to the two stream choking effect,

the solution becoming isentropic (marked by a circle),

the total pressure of the primary and mixed streams becoming equal (marked by a triangle),

the flows becoming choked after mixing.

The results were plotted in groups of constant area ratio, z, with each graph representing the solutions for a particular value of z and two values of h. It was not found practicable to plot the solutions for all values of h on one graph because of the overlapping that occurred at high values of p_{21}.

The results of this survey may be seen plotted on Figs. 8 to 21. The basic solution for fixed values of z, h and π consists of a parabolic type curve with a minimum value for p_{21}. Each point on the upper branch of the curve is a subsonic solution corresponding to a specific value of back pressure. In some solutions the parabolic curve is truncated before the minimum in p_{21} is reached. In these cases the flow chokes due to the two stream effect. The effect of increasing the total pressure ratio π is to move the whole of the basic curve up to a higher value of p_3 and in some cases slightly to the right, without changing its shape. This increases the minimum value of p_{21} in those cases that choke at the end of mixing. The value of p_{21} at which the two stream choking effect occurs rises fairly rapidly with π, and so a relatively greater portion of the basic curve is truncated with each increment in π. The point at which the primary stream becomes choked moves even faster to the right as π increases, in fact, for the values of π above the critical value, the primary stream is choked for all values of p_{21} less than P_{t2}. So the primary stream choking points may only be marked for subcritical values of π. These two choking effects are independent of the total enthalpy ratio as they both occur before the mixing process starts.

The general effects of the total enthalpy ratio on the solution may best be seen by looking at Fig. 6, where the solutions for $z = 0.5$ and $\pi = 1.7$ are plotted for the various values of h. The two stream choking effect and the choking of the primary stream are independent of h, and are represented by just two lines. The total enthalpy ratio affects the shapes of the basic solution curves but preserves their parabolic form. For high values of h the basic curve is a shallow one and has a high minimum value for p_{21}. As h falls the basic curves elongate in the negative p_{21} direction and lie completely outside curves for higher enthalpy ratios. Thus the value of p_{21} at which the mixed stream chokes falls with h until it reaches the two stream choking limit. Further reduction in the total enthalpy

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ratio causes the two values of p_3 at which the two stream choking occurs to diverge, splitting the solution into two distinct parts, one totally subsonic and the other totally supersonic. The two halves lie above and below the solutions for higher values of h respectively.

Increasing the value of z, that is increasing the primary injection area, has two main effects. It moves the basic curves up and to the right, that is, to higher values of both p_3 and p_{21}, and it also increases the value of p_{21} at which the two stream choking effect occurs, thus decreasing the possible range of p_{21}. Increasing z also increases the domain of the two stream choking effect in the π - h field. Hence some solutions, for certain values of π and h, that choke at the end of mixing for low values of z, will, for the same values of π and h but higher values of z, choke due to the two stream effect. These effects are largely due to the relative reduction in the area available for the primary expansion. The spread in the basic curves for different total pressure ratios is more pronounced at high values of z.

3.3.2 <u>The survey of choked solutions</u>

The second part of this investigation consists of a survey of the solutions in the supersonic regime. Now these solutions are all independent of the back pressure and thus the total field of possible solutions may be represented by a three dimensional field formed by the remaining control variables. The reduction in the number of control variables allows the problem to be solved over a wider range of conditions. The ranges within which solutions were found are as follows:-

z,	the geometric ratio,	from 0.01 to 0.9
π,	the total pressure ratio,	from 1.1 to 6.0
h,	the total enthalpy ratio,	from 1.0 to 5.0

As mentioned above, there are two types of solutions in the supersonic regime; those that choke after mixing and those that choke before. The boundary between these two types is the locus of solutions which choke simultaneously in both modes. This locus forms a surface in the z, π, h field which is depicted as lines of constant z in the π - h field in Fig. 22. It may be seen that the surface tends towards being plane when it coincides with the line π = h = 1. In the domain bounded by the surface and the plane h = 1 the solutions choke due to the two stream effect, and, in the domain between the surface and the plane π - 1 they choke at the end of mixing.

When the solutions are considered in terms of the state of the mixed stream, those that choke after mixing are single valued, and those that choke before are double valued. The state of the mixed stream may be represented by the exhaust pressure p_3. Looking at the solutions for a fixed geometry, they may be represented by surfaces in the π, h, p_3 field. In fact the complete solution for a fixed geometry consists of two intersecting surfaces, one corresponding to each of the choking modes. The surface representing the two stream choking solution is of a parabolic type, double valued in p_3. The solutions are shown in Figs. 23 to 28 as lines of constant total pressure ratio, π, forming the surfaces in the π, h, p_3 field. The intersection of the two surfaces may be obtained by tracing the locii of the triple points. It will

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be noticed that this line always intersects the $h = 1$ plane at the point $\pi = 1$, $p_3 = 0.5283\, P_{t2}$ (critical value for $\gamma = 1.4$) as could have been forecast from looking at Fig. 22. The solutions that choke after mixing appear to be affected only slightly by the value of h, whatever the values of z and π.

It is also of interest to look at the intersection of these solution surfaces with the plane $h = 1$. These solutions, where both streams have the same total enthalpy, are depicted in Fig. 27. As expected, all of these are double valued in p_3, as they all choke due to the two stream effect. It will be noticed that the solutions for large primary injection areas (high values of z) are limited in the size of the total pressure ratio allowed. This is because these solutions are in the base pressure regime and the secondary mass flow is reduced to zero. The value of π at which this takes place is determined by the simple and rather crude process of equating the pressure, which the primary stream would attain if it expanded isentropically to fill the whole mixing duct to the total pressure of the secondary. This can only be said to give a very approximate result as solutions in the base pressure regime are highly affected by the viscous interaction between the streams. To obtain a better result, one would have to resort to a two-dimensional method such as that of Chow and Addi, Ref. 2.

4. Extension of the Theory for the Case of Supersonic Injection

The theory developed above may be extended to cover certain flow regimes for supersonic injection. The set up referred to as a supersonic ejector in this paper is illustrated in Fig. 4a. It consists of two nozzles discharging into a constant area mixing duct. One of the nozzles is purely convergent and the other is convergent-divergent. The latter is referred to as the primary injection nozzle and the stream that passes through it is called the primary stream. To give an idea of the physical context into which any theory on supersonic injection must fit a brief description of the various flow regimes that are found to exist will be given.

Firstly there is the subsonic regime in which both streams are subsonic throughout the apparatus. The injection pressures are equal, the two stagnation pressures and the back pressure are of the same order, and the whole flow is dependent on the back pressure against which the system exhausts.

Next there are the mixed regimes. In these regimes the primary stream is choked at the throat of the primary injection nozzle, but the mixed stream is subsonic and its mass flow is therefore dependent on the back pressure. The mixed regime may be sub-divided according to the behaviour of the stream on entry into the duct. If the primary is overexpanded on entry to the duct, that is the primary injection pressure is less than the secondary injection pressure, as in Fig. 4b, then the flow is said to be in the mixed supersaturated regime. On the other hand, when the secondary injection pressure is less than the primary and the primary enters the duct in an underexpanded state the flow is in the mixed supersonic regime as in Fig. 4d. The condition between the two, when the injection pressures are equal is known as the saturated condition and is shown in Fig. 4c. In all the mixed regimes the total mass flows are always back pressure dependent.

There occurs, for any given ejection configuration, a certain limit in the back pressure below which the whole flow becomes independent of the back pressure. The regimes of the solution in which this occurs are termed the supersonic regimes and they are further sub-divided into regimes depending on the mechanism whereby they choke. It is possible if the enthalpy ratio between the two streams is very large for the mixed flow to become sonic and choked at the end of the mixing tube.

It is not proposed to deal with these cases in detail, in fact it will be assumed for the purposes of the following discussion that the total enthalpy ratios are small enough to avoid this mode of choking. The choking will therefore take place somewhere near the injection plane with the actual mechanism depending upon the reaction between the two injected streams. It is assumed that in general the choking reaction occurs before the flows have had time to mix appreciably. The sub-division of the supersonic regimes is the same as that for the mixed. When the primary is overexpanded and the secondary flow is choked the flow is said to be in the supersonic supersaturated regime (Fig. 4s). When the primary is underexpanded the regime retains the name supersonic regime (Fig. 4h). Again there is a condition between the two in which the injection pressures are equal and this is termed the saturated supersonic regime (Fig. 4b).

It is in their treatment of the supersonic regime that the various theories on supersonic ejection differ. It is commonly agreed that the secondary stream chokes by reaching sonic velocity at the throat of its injection nozzle in the saturated and supersaturated regimes. However, for the supersonic regimes where the primary stream expands on entry to the mixing tube, the secondary contracts on entry and therefore cannot reach sonic velocity at the throat of its injection nozzle. The choking therefore must occur in the mixing tube. There are two main models used to explain the mechanism of this choking. One assumes that the primary stream goes on expanding until the secondary stream reaches sonic velocity thus choking itself. The other assumes that the secondary stream chokes due to the two stream choking effect before it reaches the sonic velocity. Using either of these models of choking, there are a number of assumptions or methods of analysis which may be used to describe the exact mechanism of the choking process. For instance, the streams may be treated isentropically, or be assumed to behave polytropically, and either a one-dimensional or two-dimensional method of analysis may be used. To illustrate some of the combinations already used for supersonic injection a few examples are given.

Fabri[1], treating both streams one-dimensionally and isentropically, assumed that the secondary stream choked by reaching sonic velocity. He equated the momentum exchange between the streams but not the pressures at the end of the expansion. Messrs. Chow and Addi[2] again treated both streams isentropically and assumed that choking took place at sonic velocity, but they treated the primary stream two-dimensionally by the method of characteristics. They also include the effect of mixing between the streams before choking. Messrs. Hoge and Segars[4], on the other hand, suggest that the two stream choking effect might produce a more accurate description, though they compare its results with those assuming complete mixing and therefore are presumably talking of reasonably short ejectors. They do not seem to suggest that two stream choking is a competitor, in the strict sense of this discussion, to the sonic choking of the secondary stream in ejectors where mixing is assumed to be complete.

It is the purpose of this section to show that the two stream choking concept may well provide a better description of the flow in certain regimes. As a preliminary to the detailed discussion of the supersonic regime a description of the performance of a typical supersonic ejector is presented. The assumptions made are the same as those made earlier in the case of sonic injection.

Consider a supersonic ejector system with fixed geometry such as that shown in Fig. 4a. If the stagnation conditions are kept constant upstream, the performance of the ejector will be a single valued function of the back pressure into which the ejector exhausts. Let us consider, then, the performance of the ejector as the back pressure is lowered from a reasonably high value.

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At back pressures somewhat greater than the secondary total pressure, the ejector will operate in the subsonic regime provided that the total pressure ratio between the two injected streams is not too great. Here the flow is subsonic throughout and the injection pressure of the two streams are equal. If the total pressure ratio, π, is at all high the flow may never be subsonic throughout and still maintain a positive secondary mass flow. That is the mixed regime and the base pressure regime overlap to the exclusion of the subsonic regime. Thus, in general, at the highest back pressures to give positive secondary flows, ejectors will operate in either the mixed or subsonic regimes depending mainly upon the value of the total pressure ratio π.

For flows in the subsonic regime, upon lowering the back pressure from this maximum value, the mass flow of both streams will increase until the primary injection nozzle chokes and the flow enters the mixed regime. Further lowering of the back pressure results in the formation of a normal shock within the primary injection nozzle as in the case of the ordinary Laval nozzle. This shock moves down stream from the throat as the back pressure falls, its exact location being determined by the equality of the injection pressures. Meanwhile the lowering of the injection pressures continually increases the secondary mass flow. Eventually this normal shock will reach the injection plane and will become oblique, the injection pressures will no longer be equal and the primary stream will contract upon injection as shown in Fig. 4b. The mixed regime flows mentioned so far are all classified in the supersaturated mixed regime.

Now, at any time during the process just described, the secondary stream will become choked if its injection pressure falls to the critical value. This is most likely to occur for systems with low values of the total pressure ratio π. When the secondary stream chokes in this case and the primary stream is still underexpanded the flow is said to enter the supersaturated supersonic regime as shown in Fig. 4e.

Considering, however, those cases for which the secondary injection pressure is still above the critical value, the oblique shock gets weaker as the back pressure and injection pressures fall. During this process in which the normal shock becomes an oblique shock and grows gradually weaker, the difference between the injection pressures and the degree of the primary contraction rise to a maximum and then fall back to zero as the weak shock vanishes. This last condition, where the primary stream is neither over nor under-expanded, is known as the saturated condition and it represents the boundary between the supersaturated mixed regime and the supersonic mixed regime. For certain solutions it happens that the secondary injection pressure becomes critical simultaneously with the vanishing of the oblique shock, in which case the flow is said to choke in the saturated supersonic regime. This case is depicted in Fig. 4f.

Considering those cases for which the secondary stream is still unchoked in the saturated condition, further lowering of the back pressure will reduce the secondary injection pressure but not that of the primary. Thus the primary stream will be injected in an underexpanded state. The streams will then react so as to reduce this pressure difference; that is the primary expands and the secondary contracts as shown in Fig. 4d. This region of the solution is called the supersonic mixed regime. As the back pressure falls flows in this regime choke due to some reactive process between the two streams. Here we shall assume that this process is the two stream choking effect and that it occurs at the end of the primary expansion. This event is depicted in Fig. 4h.

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At this stage in a superficial investigation it might be considered that all the possible configurations of the flows had been discussed. On closer investigation, however, there appear some flows in the regime of the solution between the supersaturated supersonic regime and the supersonic regime which, when two stream choking is assumed, are in need of some explanation.

It may be instructive to look at these solutions in the following way. Consider, as above, a supersonic ejector of fixed geometry exhausting straight into a vacuum and thus operating entirely in the supersonic regime. Now consider the effect of varying the total pressure ratio, π. For extremely high values of π the ejector will be operating in the base pressure regime; that is there will be no secondary flow. For slightly lower values, the flow will be in the supersonic regime and the flow will choke due to the two stream effect. As the total pressure ratio is lowered the degree of primary expansion will decrease and the plane of two stream choking will approach the injection plane. When the expansion vanishes, the two streams will be injected at equal pressures and the choking effect will take place immediately. In this case the secondary injection pressure is well above the critical value. Now, if we are altering the value of π by altering the primary total pressure and keeping that of the secondary constant, we may identify a particular value of the primary total pressure, say P_t', with the vanishing of the primary expansion.

Now, if the problem is also tackled from the other and, that is starting with low values of π and then increasing them, it may be seen that the primary will start by being highly overexpanded, and the secondary, being choked at the injection plane, will expand considerably on entering the duct. As the value of π is increased or as the primary total pressure is increased, the primary injection pressure will also increase, eventually rising to the same level as the secondary injection pressure, which in this case is the critical pressure since the secondary stream is injected sonically. Again we may associate a particular value of primary total pressure with the equality of the injection pressures. This value, P_t'', is different to P_t', mentioned above, and the common value of the injection pressures is also different to that associated with P_t'. In the first case the secondary injection pressure is that associated with two stream choking and is therefore subsonic, but in the second case it corresponds to the critical or sonic value. There thus emerges a region within the supersonic regime bounded by the two values of the primary total pressures P_t' and P_t'' at which the injection pressures become equal. This region we shall refer to as the saturated supersonic regime.

The area of the solution involved is shown in Fig. 30, where the values of the total pressure ratio π corresponding to P_t' and P_t'' are plotted against the primary injection Mach number and the geometric parameter z. z in this case is the ratio of the area of the primary throat to that of the mixing tube. Now the solutions for the sonic choking of the secondary injection nozzle are independent of z. Hence the solutions corresponding to P_t'' may be depicted as a single curve in Fig. 30 (the curve nearest the M_{11} axis). The two stream choking solutions, on the other hand, are functions of z and are therefore represented by a set of curves, forming a surface in the π, z, M_{11} field. The solutions for high values of z are shown to intersect with the base pressure regime, as the mass flow of the secondary stream tends to zero. Thus it may be seen that solutions defined by points within the volume bounded by the surfaces of Fig. 30 will choke in the saturated supersonic regime. Those contained between the plane $\pi = C$ and the surface corresponding to P_t''

choke/

choke in the supersaturated supersonic regime. Solutions corresponding to the supersonic regime are contained in the volume bounded by the P_t' surface and the base pressure surface.

Assuming the theories proposed above for the supersonic and supersaturated supersonic regimes are applicable up to the boundaries corresponding to P_t' and P_t'' respectively, there remains a need to produce a theory to cover solutions in the saturated supersonic regime, between P_t' and P_t''. Two main possibilities emerge that seem to merit discussion, firstly there could occur a discontinuity in the secondary mass flow as the two stream choking process vanishes, and both flows could accelerate until the secondary chokes at sonic velocity. Or secondly both streams could decelerate, the primary via an oblique shock, and the secondary would choke due to the two stream effect. Both possibilities will now be discussed in full with reference to Fig. 5, where the solutions they indicate are sketched. Fig. 5 consists of a plot of the secondary injection Mach number plotted against the ratio of the secondary injection pressure p_{21} to the primary total pressure P_{t1}. The curve AB represents the two stream choking solution for the supersonic regime, and the point B, corresponding to the primary total pressure P_t', is on the line of equal injection pressures. The supersaturated supersonic regime is represented by the line DF, D being the boundary point with the saturated supersonic regime.

Consider firstly the sonic choking theory. In this case, as the primary total pressure decreases past the value P_t' the two stream choking process vanishes and the secondary mass flow is suddenly allowed to increase so that the secondary injection pressure suddenly drops. This discontinuity occurs at a particular value of total pressure ratio and therefore may be represented on Fig. 5 as a line of constant total pressure ratio, say BC. The primary stream then expands until it chokes the secondary. It is this choking process that now governs the secondary injection pressure, thus defining the point C. As the total pressure ratio is lowered the degree of primary expansion decreases and when eventually it vanishes the secondary stream chokes at the plane of injection. This solution is represented by the curve BCD in Fig. 5.

Alternatively, there is the two stream choking theory. As the primary total pressure falls below the value P_t', this theory requires no discontinuity in the secondary mass flow but it does require the primary stream to contract when its injection pressure is higher than that of the secondary. The most likely possibility is for the primary to pass through an oblique shock system and this would allow the secondary stream room to expand subsonically until the two stream choking effect occurs. This solution forms a natural extension to the curve AB in Fig. 5 and extends up to meet the extension of the line FD at E where the secondary chokes in the plane of injection. The solution follows the curve BE as the total pressure ratio decreases from P_t'. The degree of the primary contraction and the strength of the oblique shock increase until the point E is reached whereupon they both start to subside again as the curve ED is traversed. At D the shock and contraction have both vanished and the injection pressures are again equal.

Two theories have been presented to cover the flow in the saturated supersonic regime. Which, if either, will give an accurate description of the flow within this region of the solution is not known at present. The few odd experimental points that have been published for this regime seem to favour the two stream choking explanation, but it is not really possible to come to any definite conclusions without carrying out specific experiments designed especially for the purpose.

5./

5. Conclusion

An attempt has been made to find an improved one-dimensional theory to predict the performance of constant area ejectors with convergent injection nozzles. Theories for the operation of such ejectors in the subsonic, mixed and supersonic regimes have been presented. They are all based on the assumption that the flows may be divided up into specific regions in which the processes of mixing and pressure adjustment may take place separately. The theory covering the supersonic regime assumes that only two modes of choking occur, one at the exit of the mixing duct and the other due to the interaction of the two injected streams when injected at different static pressures. The domains over which each mode is active have been outlined. The two stream choking effect is assumed to take place at the end of the primary expansion. It may be possible, however, that such an effect takes place during the mixing process, thus providing a gradual transition between the two extreme causes of choking assumed in the theory above. The theory is not applicable to the base pressure solutions; that is those with zero or near zero secondary mass flows. To obtain satisfactory solutions in this regime it is considered necessary to adopt to two-dimensional approach that takes into account the viscous interaction between the two streams.

From the discussion of the supersonic solutions it is concluded that the ones that are most likely to exist physically are those corresponding to maximum mass flow solutions in which the system chokes due to the two stream effect. All other solutions will remain at maximum flow conditions in which the mixed stream becomes choked. Hence it is also concluded that the total pressure of the mixed stream always lies between those of the two injected streams. All such solutions are found to be irreversible, that is they produce entropy.

It has been shown how these theories may be adapted to include supersonic injection and the mixing of different gases. In the case of supersonic injection it is found that the two stream choking effect is applicable to those flows in which the primary stream is injected in an underexpanded state.

The results predicted by this approach have not yet been compared fully with any experimental work. This is mainly due to the lack of any published results of a suitably comprehensive nature. The author, at the time of writing, is about to embark on an experimental programme.

References/

References

No.	Author(s)	Title, etc.
1	J. Fabri and J. Paulon	Theory and experiments on supersonic air-to-air ejectors. NACA TM 1410. September, 1958.
2	W.L. Chow and A.L. Addi	Interaction between primary and secondary streams of supersonic ejector systems and their performance characteristics. AIAA Journal, Volume 2, No. 4. April, 1964.
3	H. Pearson, J.B. Halliday and S.F. Smith	A theory of the cylindrical ejector supersonic propelling nozzle. Journal of the Royal Aeronautical Society, Volume 62, p.746. 1958.
4	H.J. Hoge and R.A. Segars	Choked flow: a generalisation of the concept and some experimental data. AIAA Journal, Volume 3, No. 12. December, 1965.

Appendix I/

Appendix I

Derivation of the Mass Flow Density and Momentum Flow Functions

The mass flow, $\dot{m}$, of a perfect gas flowing steadily past a certain cross-section in a duct, is given one-dimensionally by the following expression:-

$$\dot{m} = A \rho v$$

where A is the duct cross-section,
ρ is the density of the gas,
and v the velocity of the gas.

Introducing the perfect gas law,

$$p = \rho R T$$

where p is the pressure,
R the gas constant,
and T the temperature,

and using the normal isentropic relations the expression for the mass flow may be rewritten,

$$\begin{aligned} \dot{m} &= A \rho v \\ &= A \tilde{w} P_t \, a \, M/(R\,T) \\ &= \frac{A P_t}{\sqrt{R\,T}} \cdot \sqrt{\frac{2\gamma\,(1 - \tilde{w}^{(\gamma-1)/\gamma})}{\gamma - 1}} \cdot \tilde{w}^{1/\gamma} . \end{aligned}$$

where M is the Mach number,
T_t the total temperature,
P_t the total pressure,
γ the ratio of the specific heats,
$\tilde{w}$ the isentropic pressure ratio $\dfrac{p}{P_t}$

Now if we define the mass flow density function y as the function $\left(1 - \tilde{w}\,\dfrac{\gamma - 1}{\gamma}\right)^{\frac{1}{2}} \tilde{w}$ of $\tilde{w}$, we see that the mass flow may be written:-

$$\dot{m} = \frac{A P_t}{\sqrt{T_t}} \cdot \sqrt{\frac{2\gamma}{R\,(\gamma - 1)}} \cdot y\ (\tilde{w})$$

Similarly the energy flow may be written,

$$\dot{m}\, Cp\, T_t = A\, P_t\, T_t^{\frac{1}{2}} \cdot Cp \sqrt{\frac{2\gamma}{R(\gamma - 1)}} \cdot y\ (\tilde{w}) .$$

The/

The momentum flow, $\dot{M}$, is given by,

$$\dot{M} = A(p + \rho v^2) \quad .$$

This may also be written in terms of the stagnation conditions and a single property which in this case we shall call the momentum flow function $f(\tilde{w})$.

$$\dot{M} = A(p + \rho v^2) \; ,$$

$$= A\left(P_t \tilde{w} + P_t \tilde{w} M^2 \right) ,$$

$$= A P_t \cdot \frac{\gamma + 1}{\gamma - 1} \cdot \tilde{w}\left(\frac{2\gamma}{\gamma + 1} \tilde{w}^{(1 - \gamma)/\gamma} - 1 \right) ,$$

$$= A P_t \frac{\gamma + 1}{\gamma - 1} f(\tilde{w}) \; .$$

where $f(\tilde{w})$, the momentum function, is defined as:

$$f(\tilde{w}) = \tilde{w}\left(\frac{2\gamma}{\gamma + 1} \tilde{w}^{(1 - \gamma)/\gamma} - 1 \right) .$$

Appendix II/

Appendix II

Extension of Theory to the Case of Mixing Two Different Gases

The theory developed above is also applicable when the compositions of the two streams are not the same. However the equations presented with this theory have to be rewritten in a more general form in order to take into account the differences in properties. Each stream will have its own values of gas constant, R, and ratio of specific heats, γ. The values of these constants for the mixed stream will depend upon the ratio of the primary and secondary mass flows and thus may only be determined when these are already known. The mass flow density and momentum flow functions must be redefined as functions the two variables $\tilde{w}$ and γ.

The mass flow past a cross-section now becomes,

$$\dot{m} = A\, P_t\, (R\, T_t)^{-\frac{1}{2}}\, y(\tilde{w}, \gamma).$$

where $$y(\tilde{w}, \gamma) = \left(\frac{2\gamma}{\gamma - 1} \left(1 - \tilde{w}^{(\gamma - 1)/\gamma}\right) \right)^{\frac{1}{2}} \tilde{w}^{\frac{1}{\gamma}} .$$

Similarly the energy flow,

$$\dot{e} = A\, P_t\, Cp\, (^{T}t/R)^{\frac{1}{2}}\, y\, (\tilde{w}, \gamma).$$

and the momentum flow,

$$\dot{M} = A\, P_t\, f(\tilde{w}, \gamma).$$

where f is redefined as

$$f(\tilde{w}, \gamma) = \tilde{w} \left(\frac{2\gamma}{\gamma - 1}\, \tilde{w}^{(1 - \gamma)/\gamma} - \frac{\gamma + 1}{\gamma - 1} \right).$$

If the mass flows of the primary and secondary streams are m_1 and m_2 respectively, the gas constant R_3 for the mixed stream will be given by:-

$$R_3 = \frac{m_1 R_1 + m_2 R_2}{m_1 + m_2}.$$

The ratio of the specific heats γ_3 for the mixed stream will be given by

$$\gamma_3 = \frac{m_1\, Cp_1 + m_2\, Cp_2}{m_1\, Cv_1 + m_2\, Cv_2}.$$

To/

To illustrate how the equations may be modified an example will be given. Consider Eqn. (9a) from section 2.2. This states that the mass flow of the mixed stream must be equal to the combined mass flows of the primary and secondary streams. It will now have to be written,

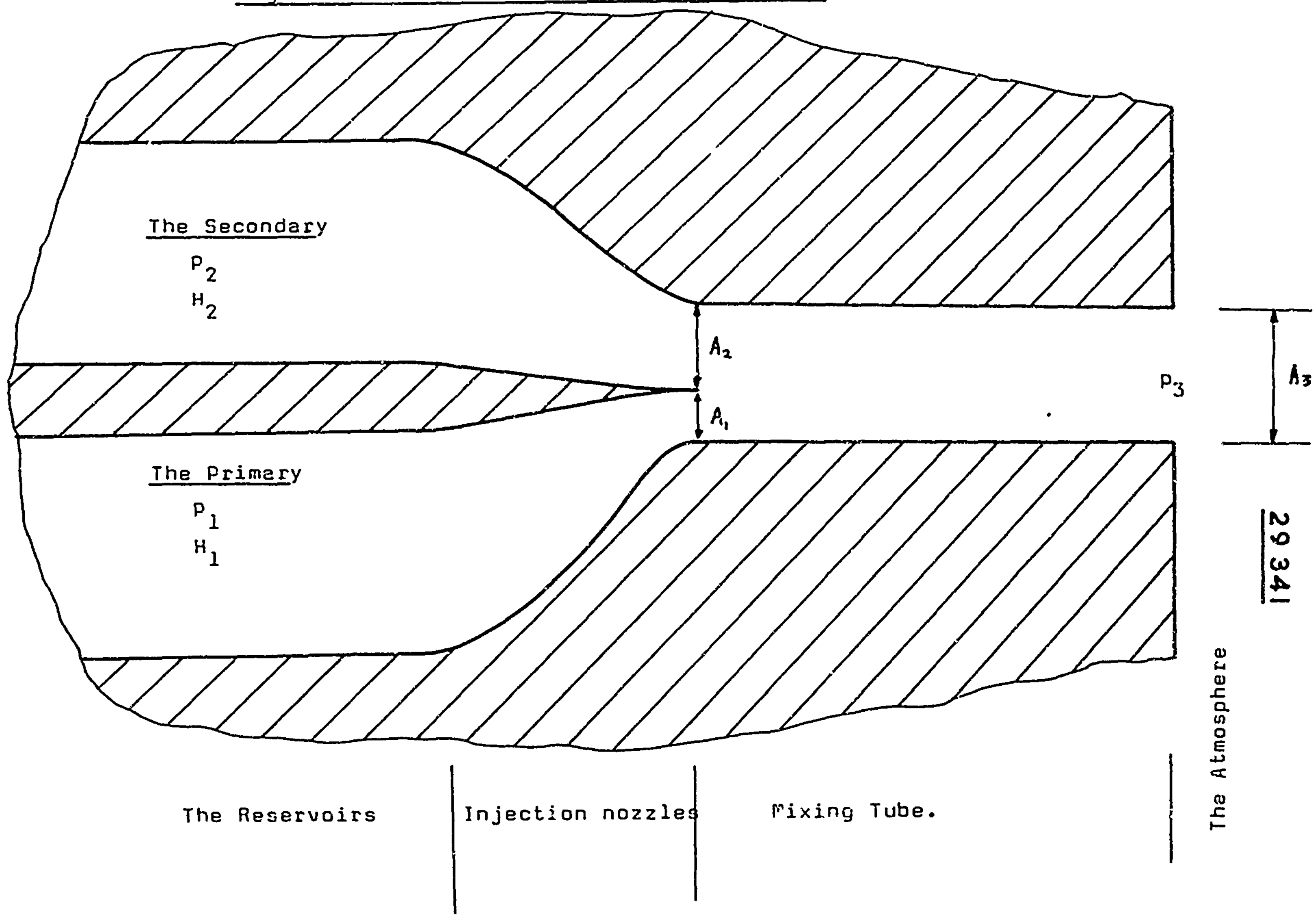
$$y(w_{22},\gamma_2) + z'' \pi' (r\ h)^{-\frac{1}{2}} y(w_{12},\gamma_1) = (1 + z'') \pi'' (r_e\ h_3)^{-\frac{1}{2}} y(w_{33},\gamma_3).$$

where $$r = R_1/R_2 \quad \text{and} \quad r_e = \frac{m_1 r + m_2}{m_1 + m_2} .$$

Having rewritten Eqns. (7c), (8c), (10a), (11a), etc., in a similar manner the same method of solution as outlined above in section 2.3 may be used to find the state of the mixed stream.

MH

Figure 1. The Constant Area Mixing System.



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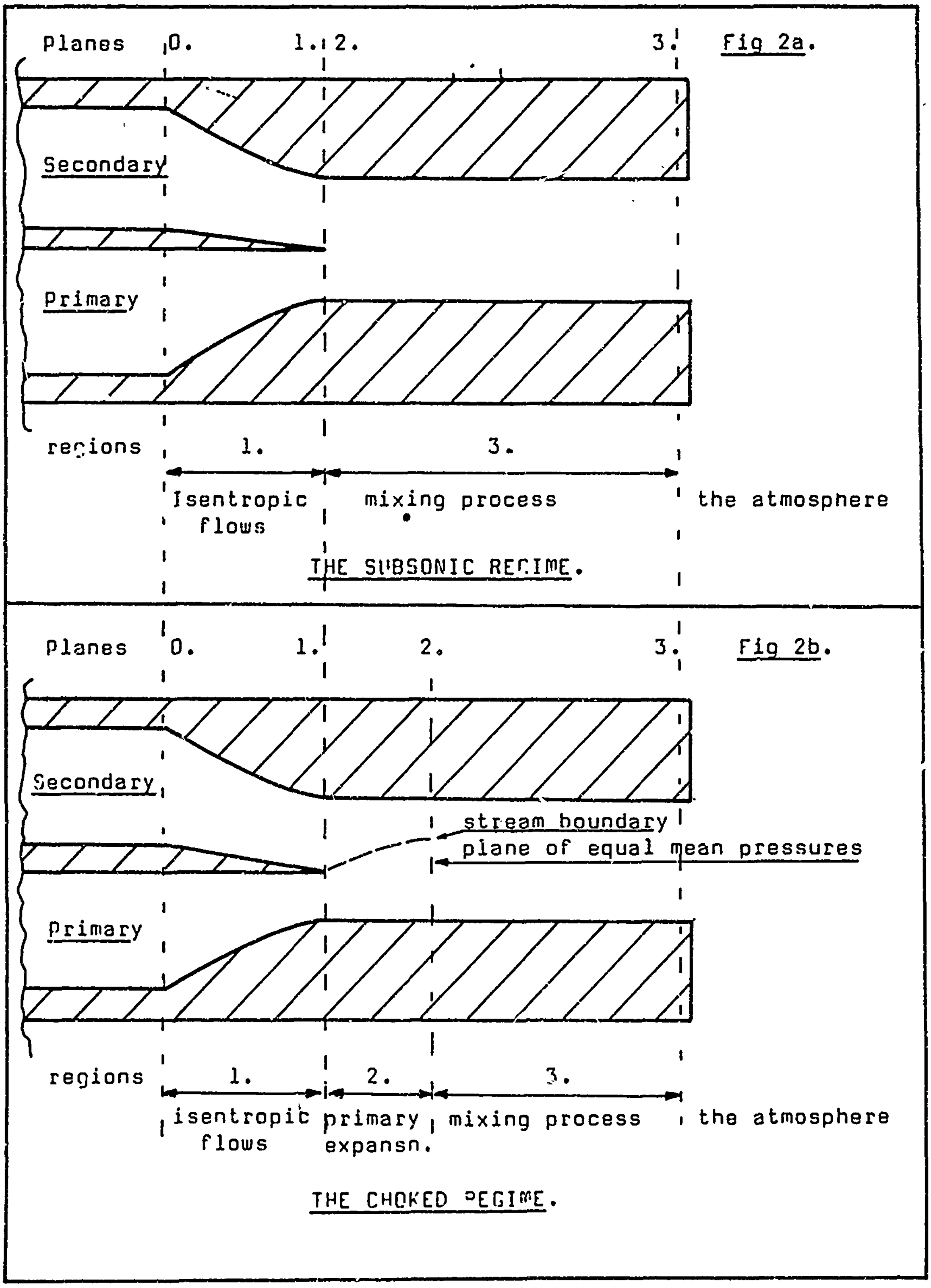

Figures 2a & 2b.

Depicting the subsonic and the choked regimes.

The regimes of the sonic ejector

FIG 3 (a & b)

Figure 3a.

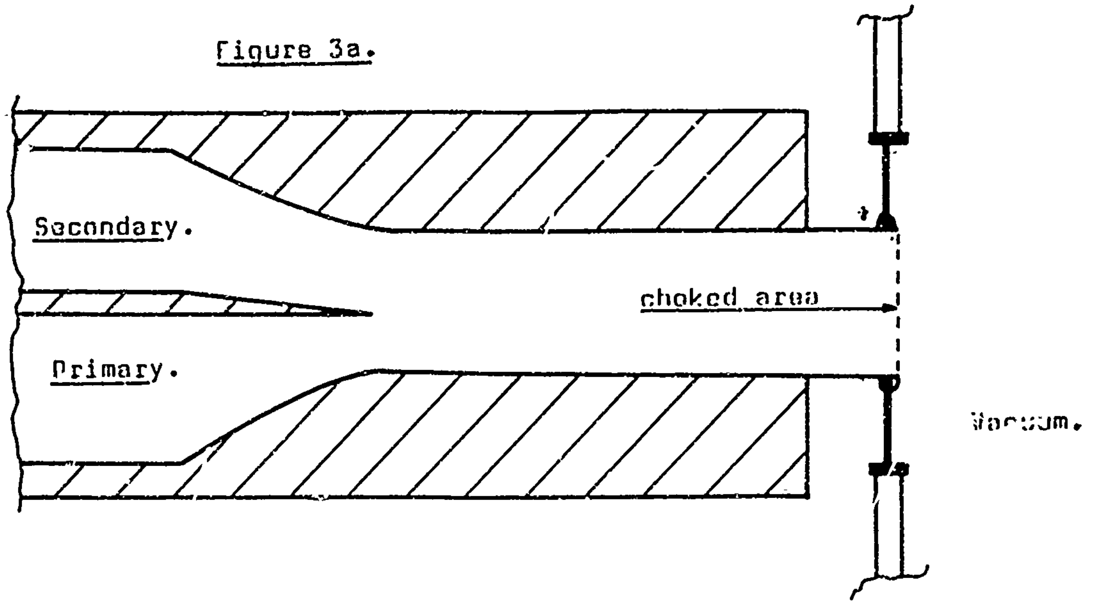

Figure 3b.

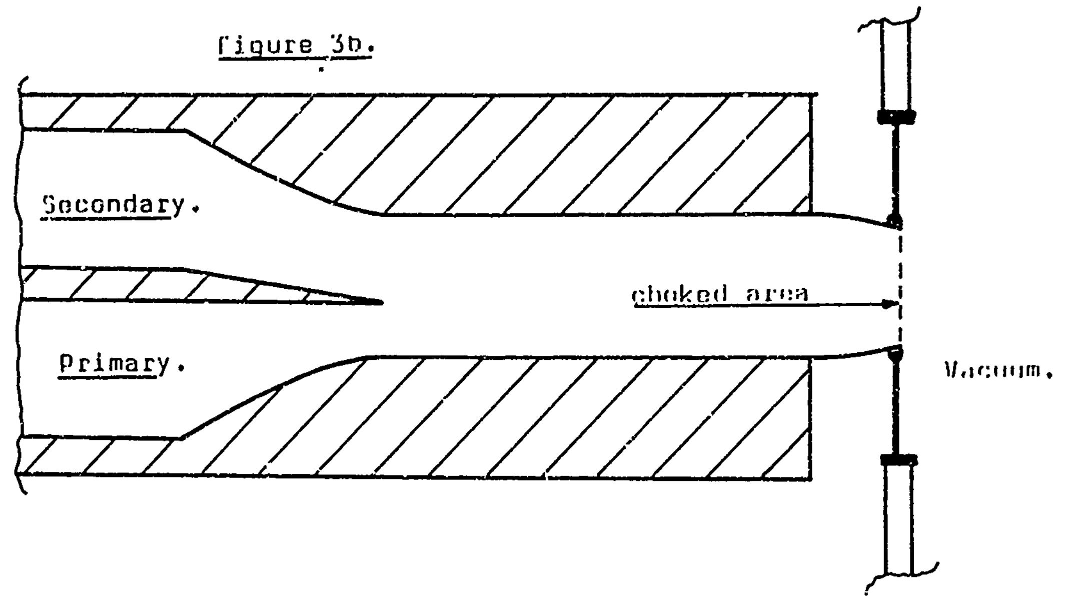

Figure 3.

Depicting a suggested method of obtaining a supersonic solution.

Supersonic solutions

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FIG. 4

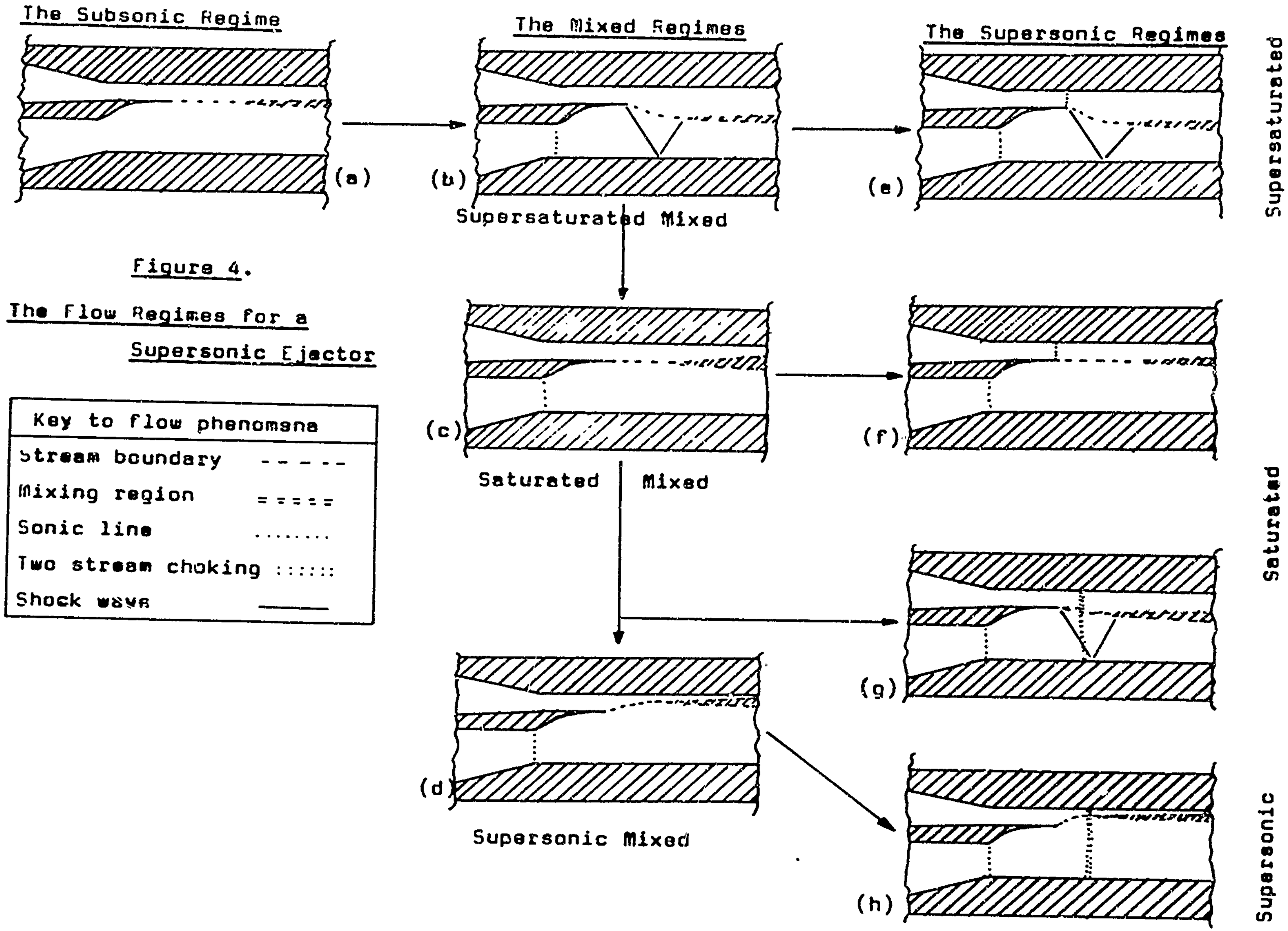


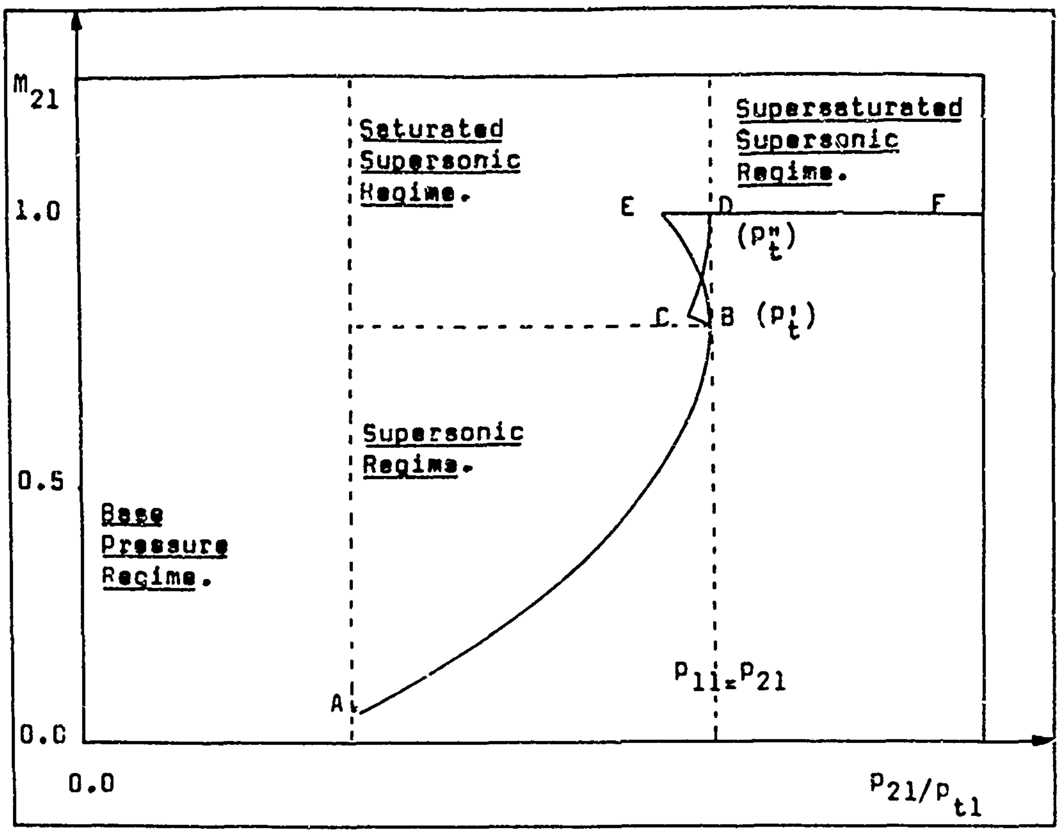
Figure 4.

The Flow Regimes for a Supersonic Ejector

Key to flow phenomena	
Stream boundary	- - - - -
Mixing region	= = = = =
Sonic line	....
Two stream choking	::::::
Shock wave	———

Figure 5. The Supersonic Regimes for a Fixed Geometry.

Secondary injection Mach number v. Secondary injection pressure/ Primary total pressure.



Figures 6 to 21.

Full Solutions for Sonic Injection.

In figures 6 to 21 a series of full solutions for sonic injection constant area mixing is given. Each curve represents the performance of an ejector of fixed geometry and fixed upstream stagnation conditions. Different points on the same curve correspond to different mass flows and different exhaust pressures. The solutions are plotted in the P_{21} - P_3 field. P_{21} is the secondary injection pressure and it is this pressure that determines the mass flows within the system. P_3 is the exhaust pressure at the exit of the mixing tube. In the subsonic solutions P_3 is also equal to the back pressure into which the ejector exhausts. Both P_3 and P_{21} are normalised by dividing them by the secondary total pressure. Points, on the supersonic branches of the solutions, beyond which there is a loss of entropy are marked thus ● and those beyond which there is a gain in total pressure are marked thus ▲ . The general form of a typical solution curve is shown in the figure below.

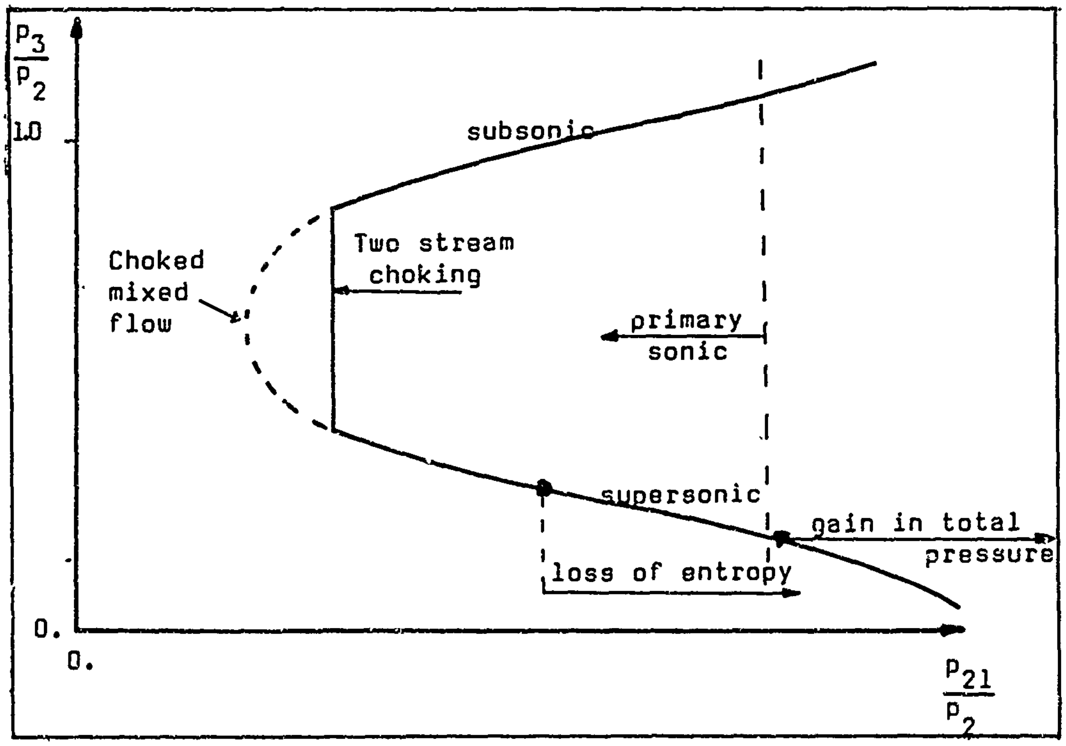


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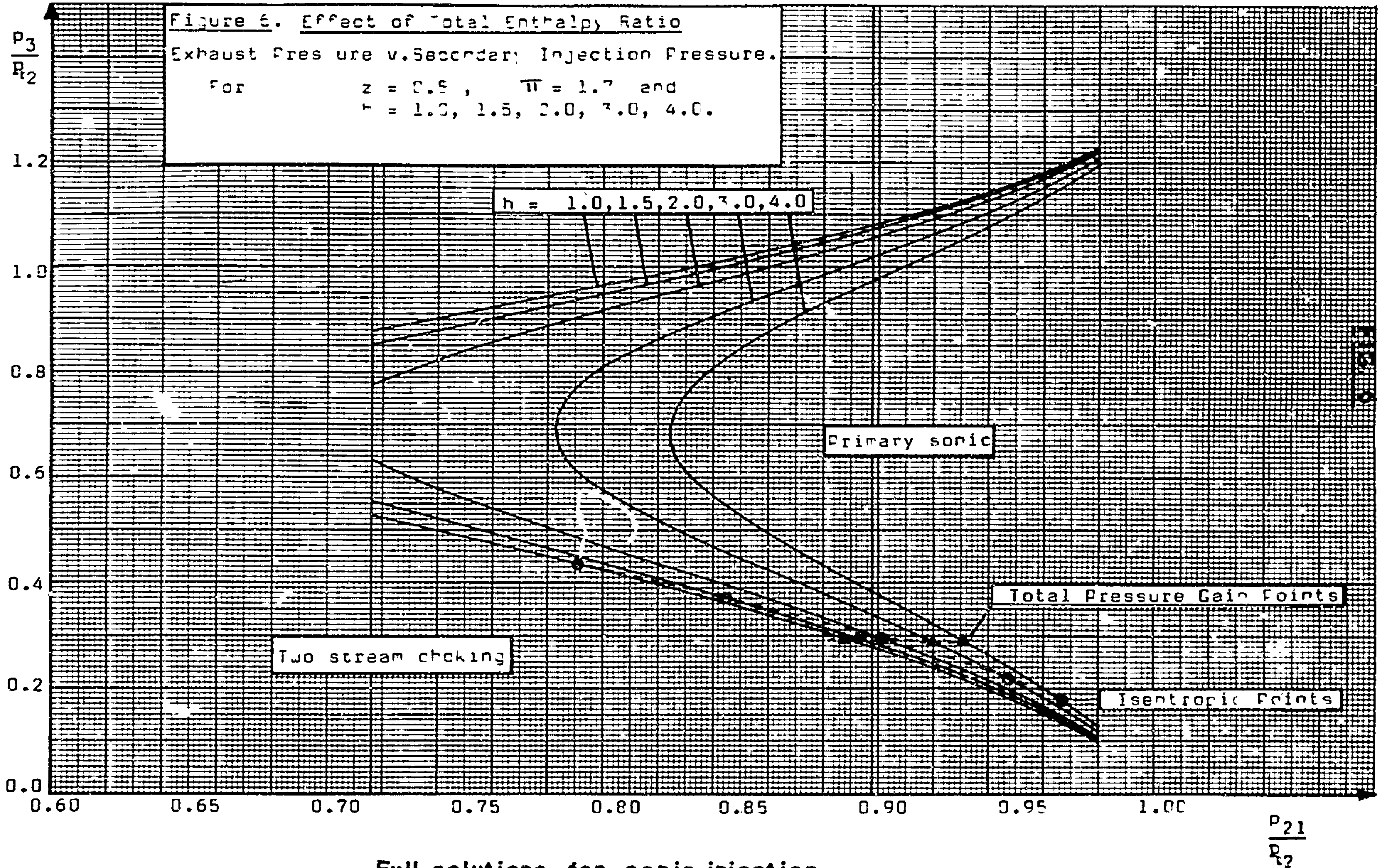
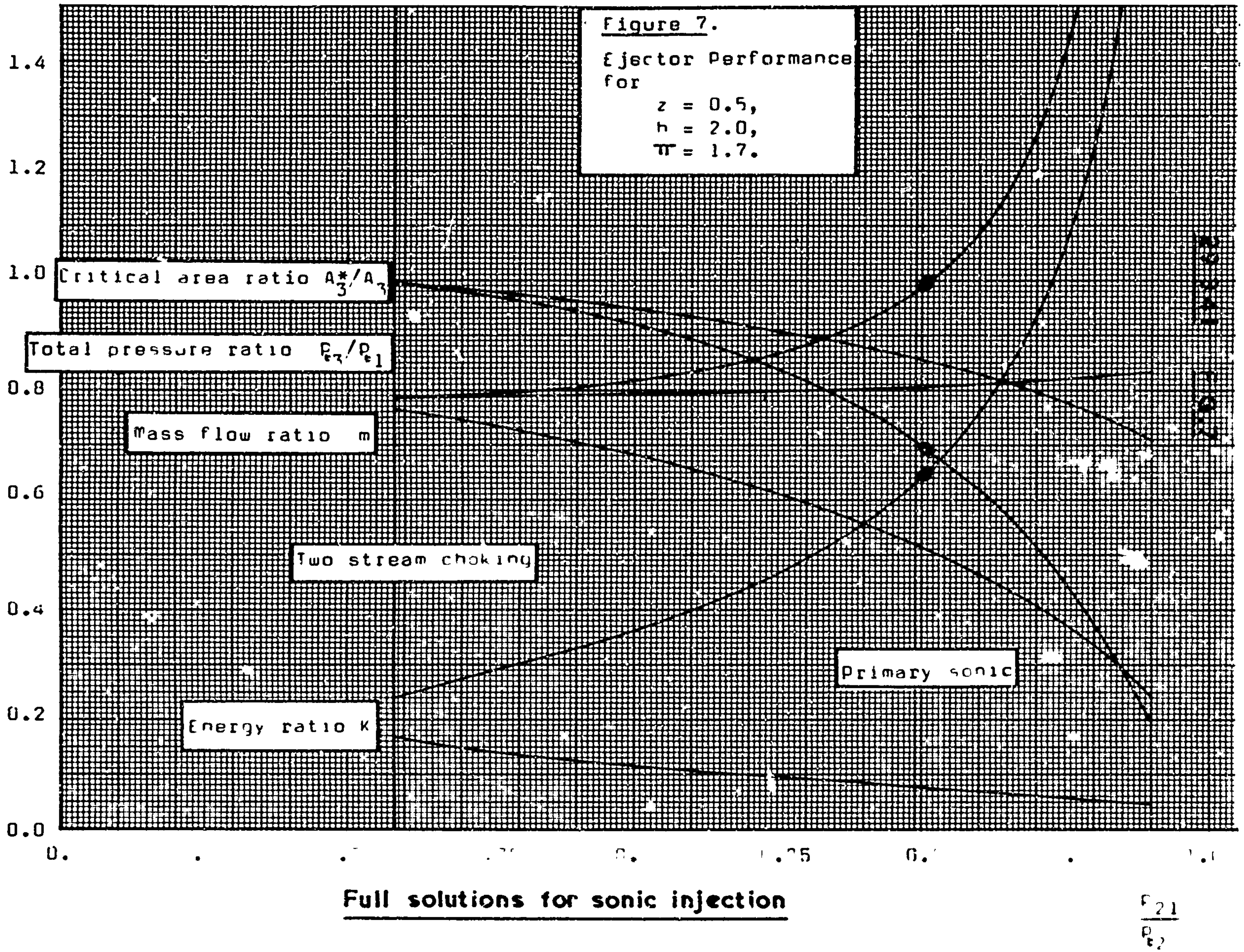
FIG. 6



Full solutions for sonic injection



Full solutions for sonic injection

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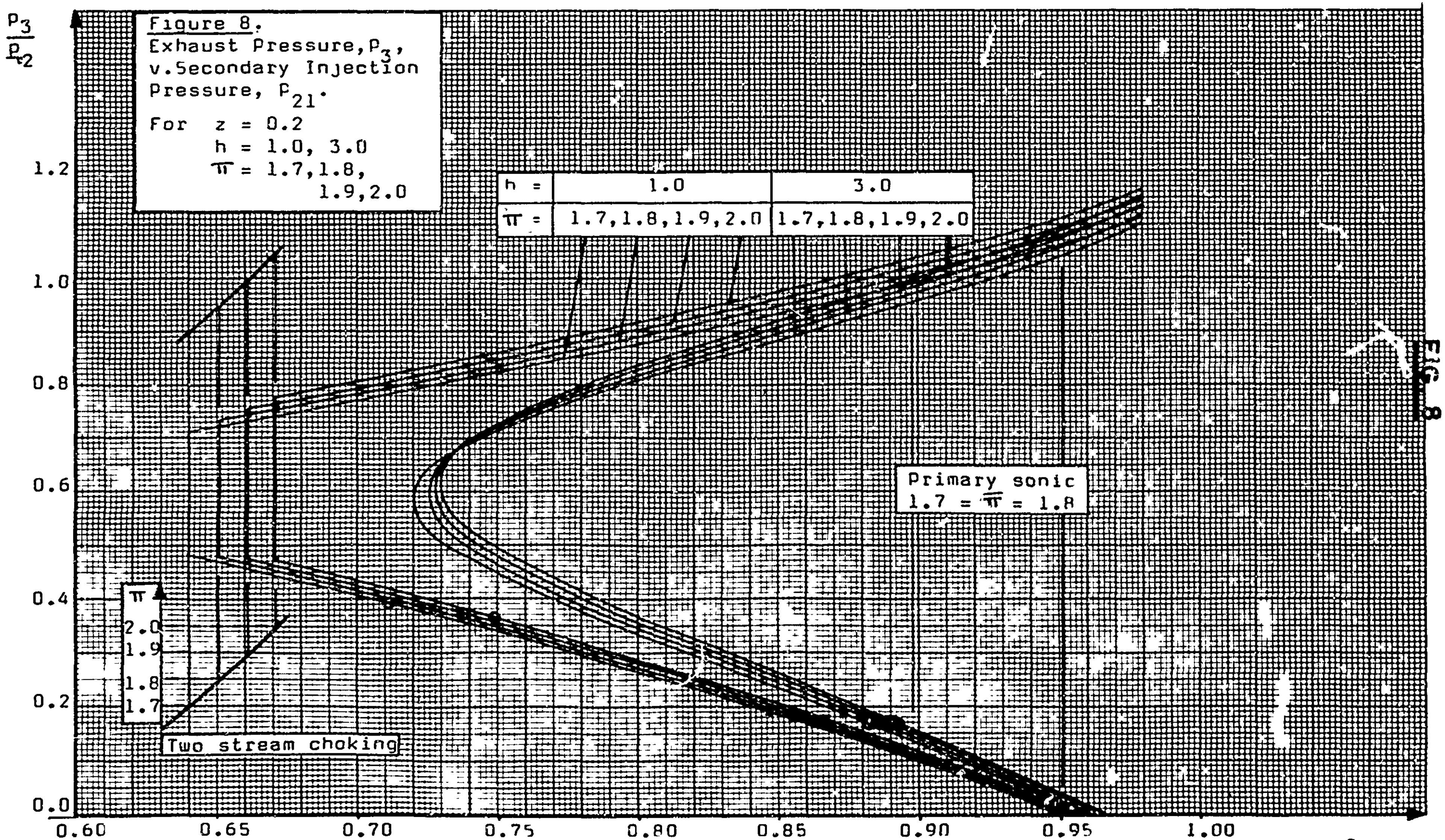
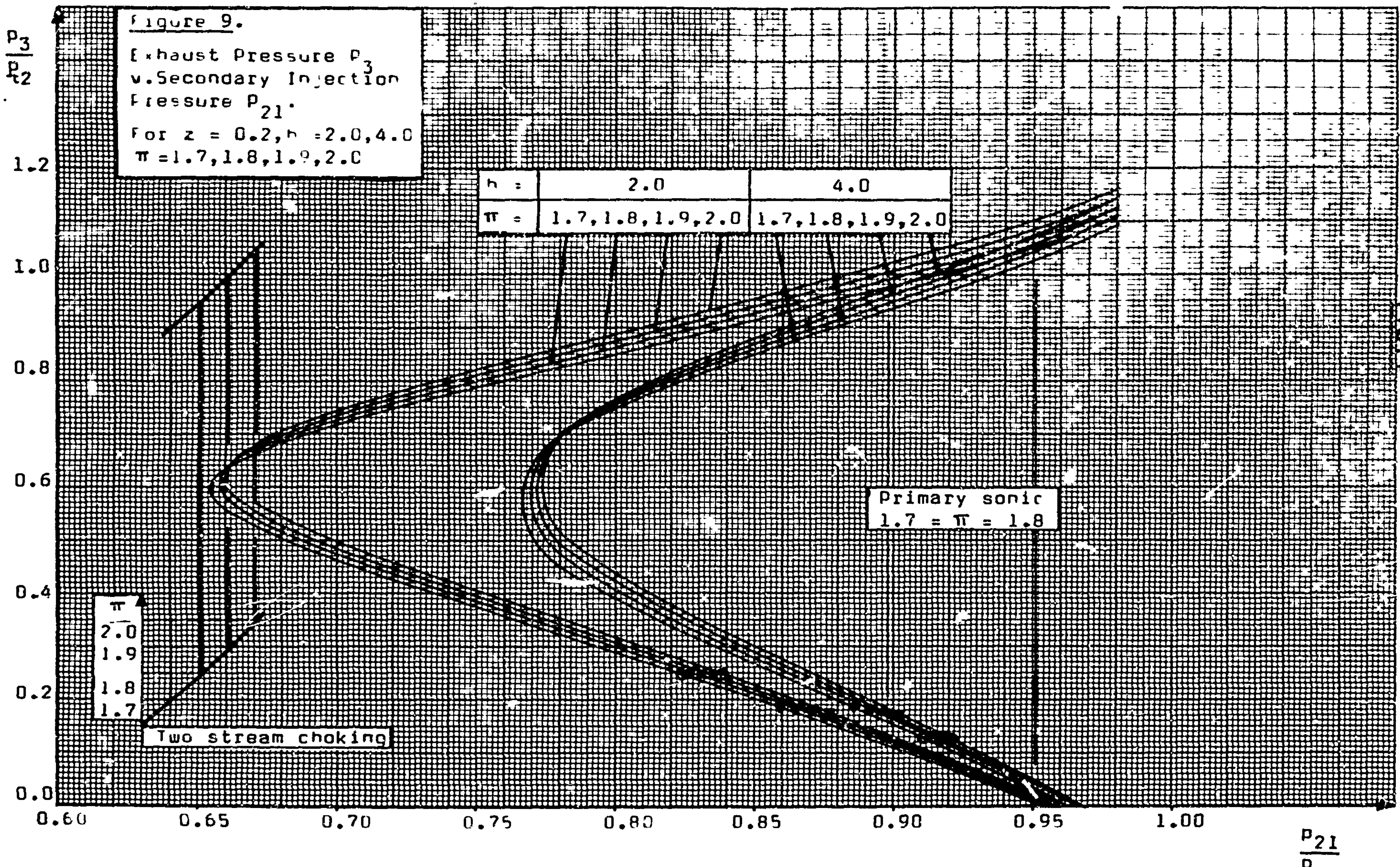
FIG. 8



Full solutions for sonic injection

29 341
FIG. 9



Full solutions for sonic injection

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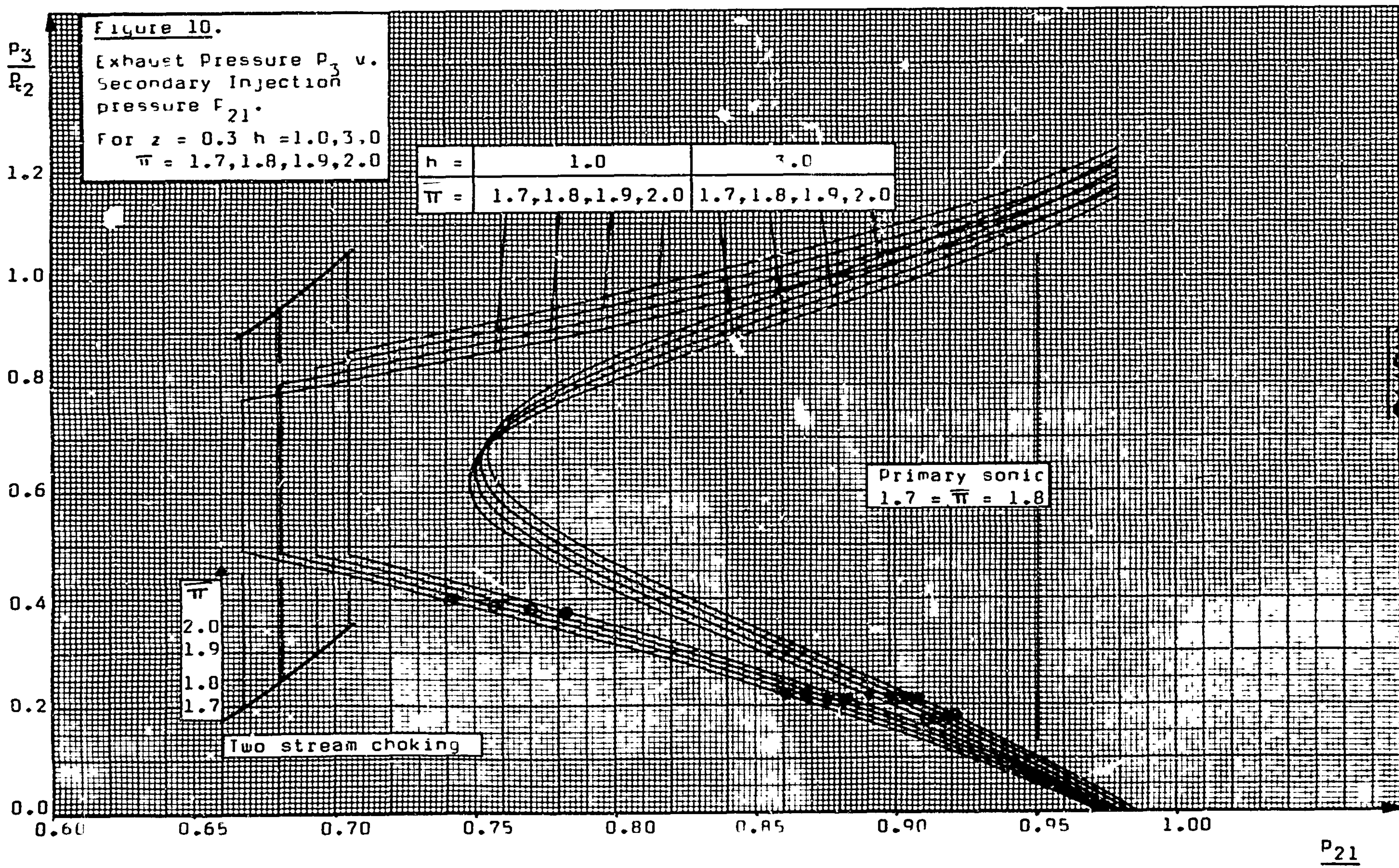
FIG. 10



Full solutions for sonic injection

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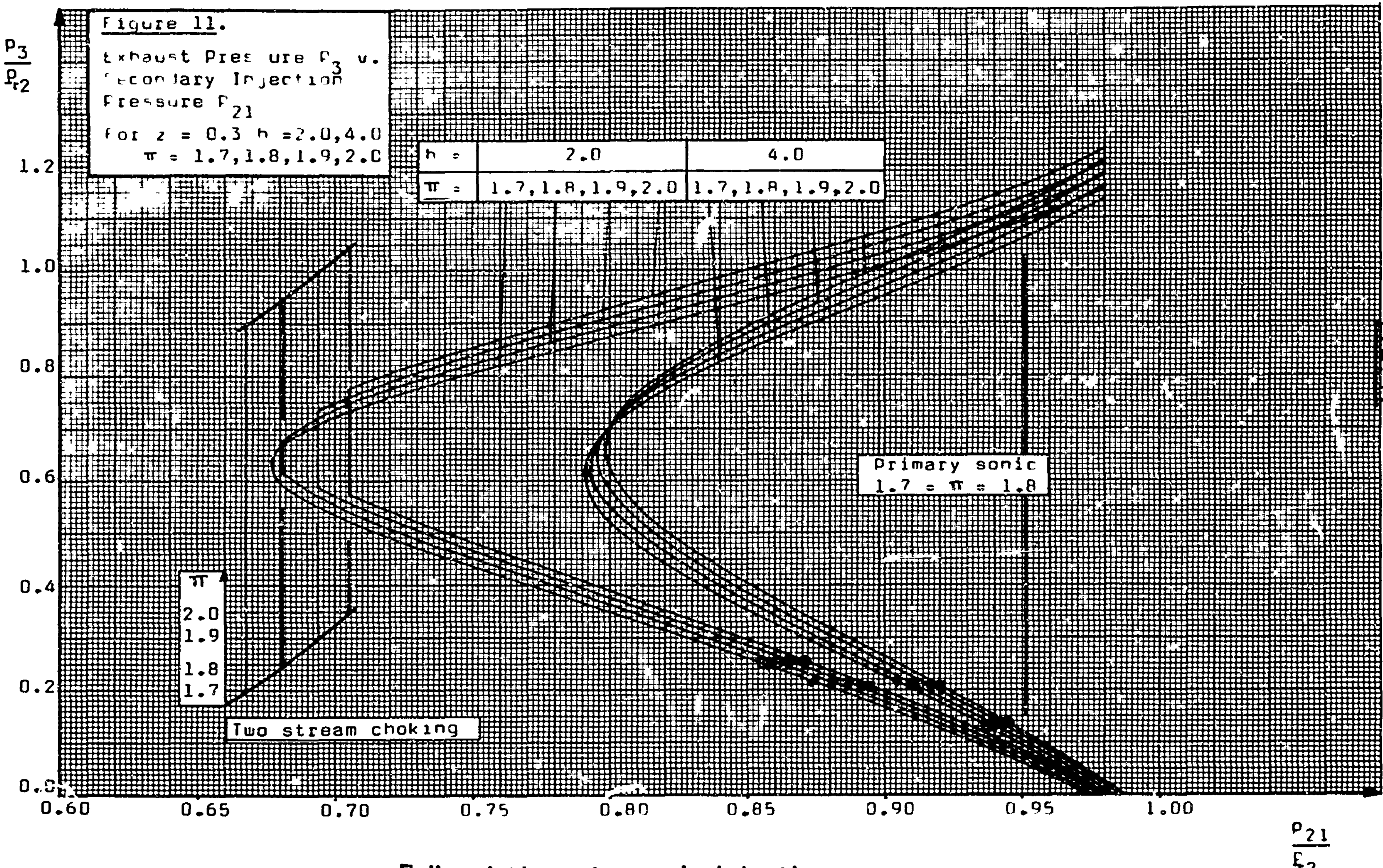
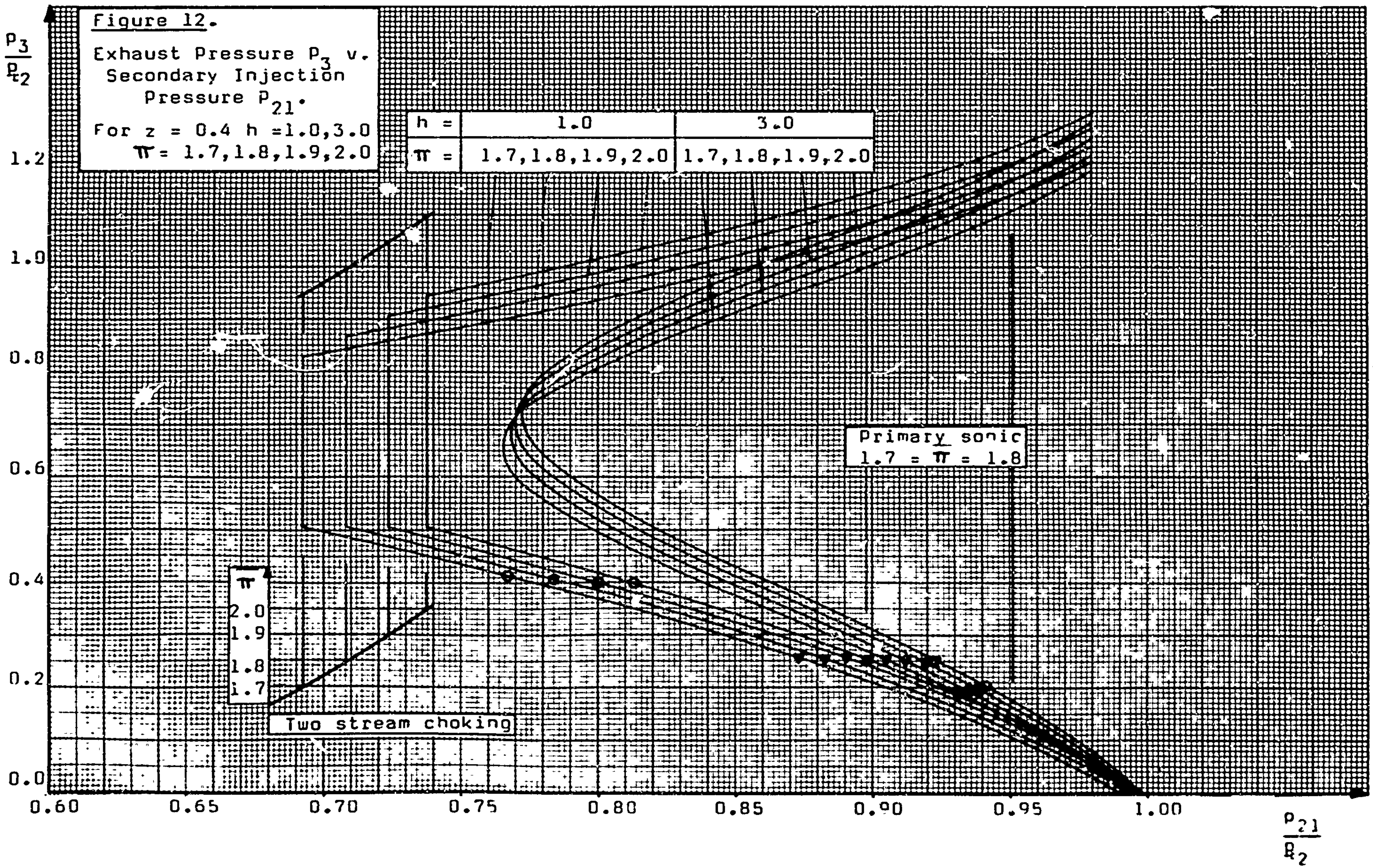
FIG 11

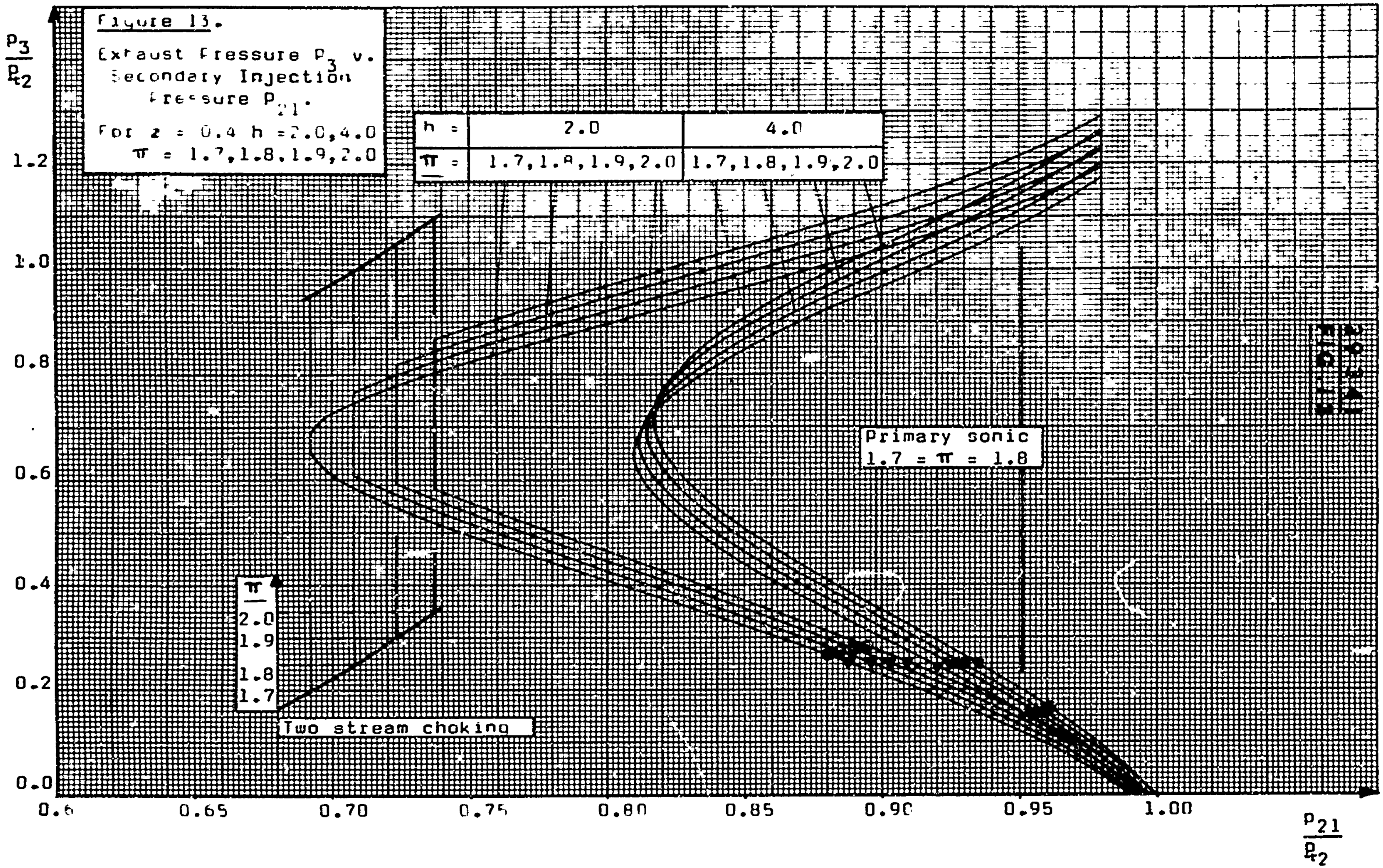


Full solutions for sonic injection

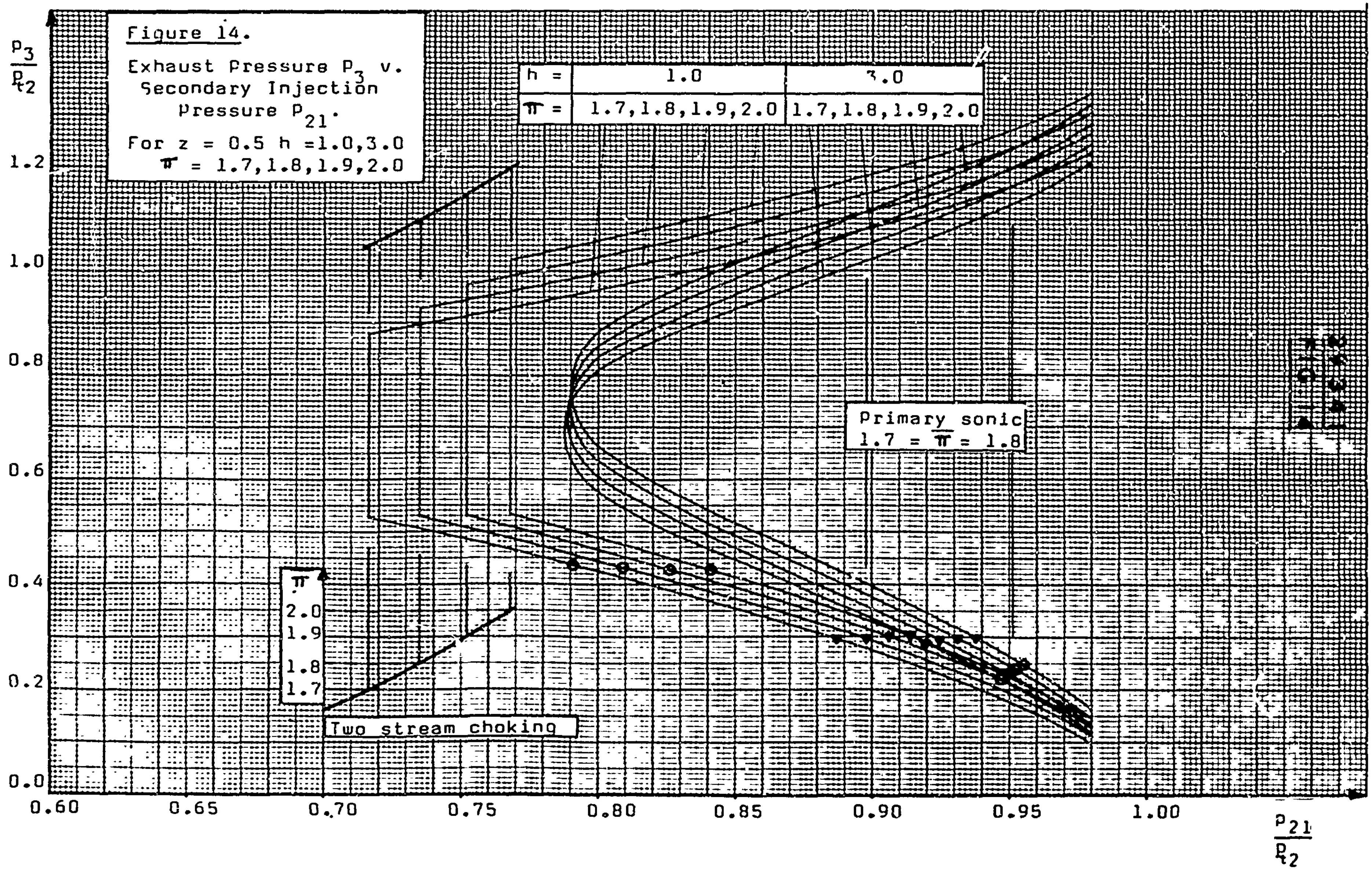
2.9341
FIG.12



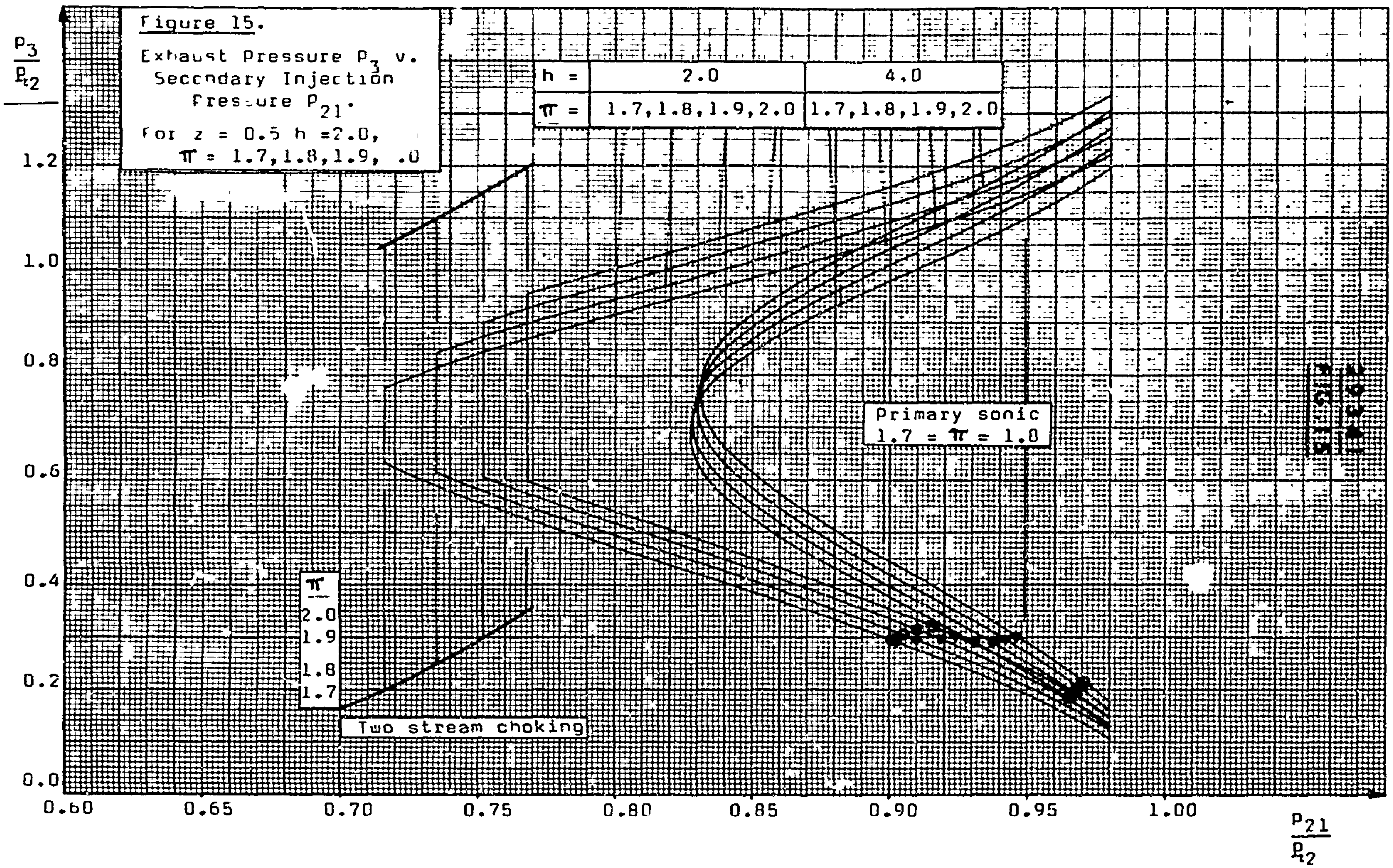
Full solutions for sonic injection



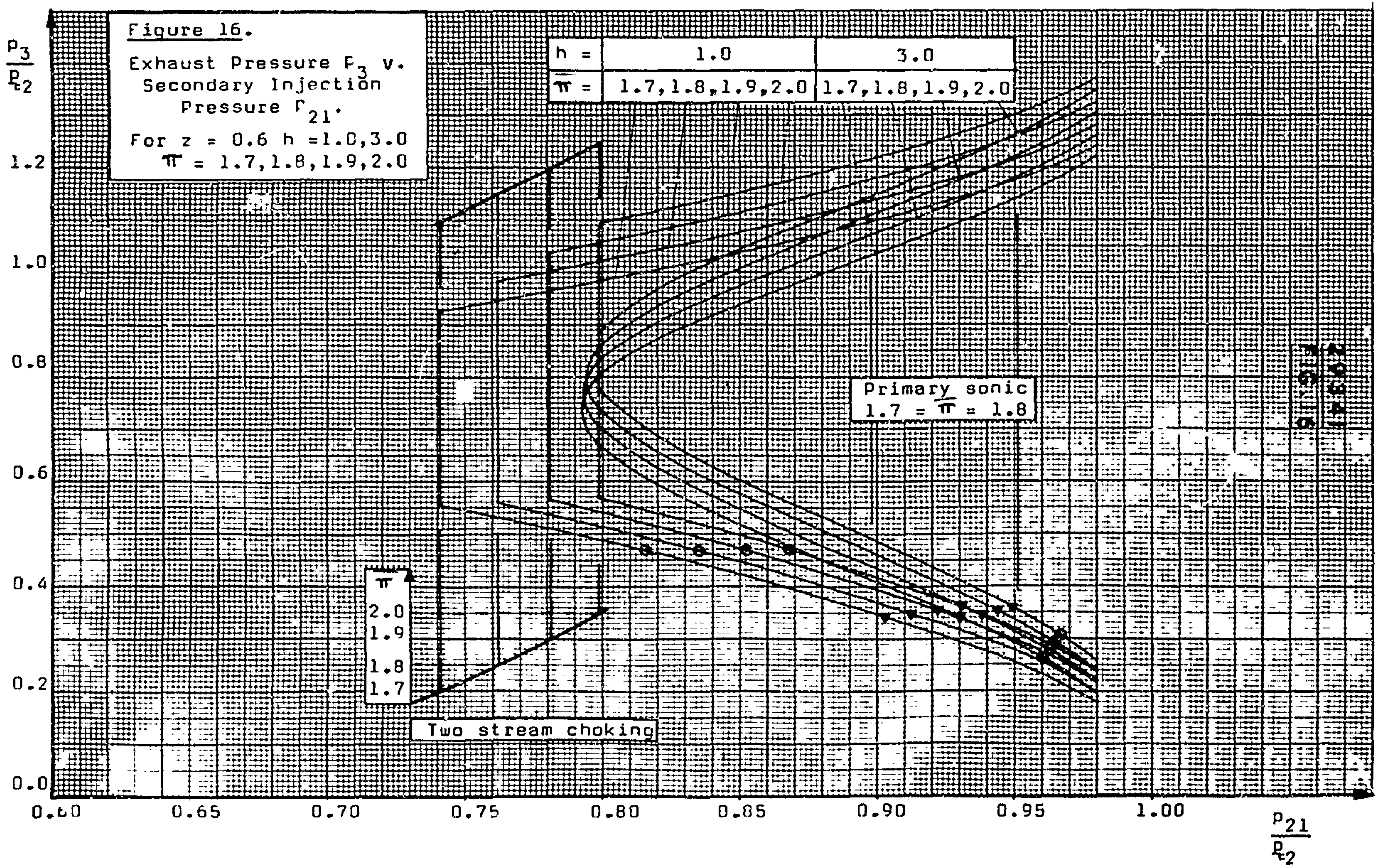
Full solutions for sonic injection



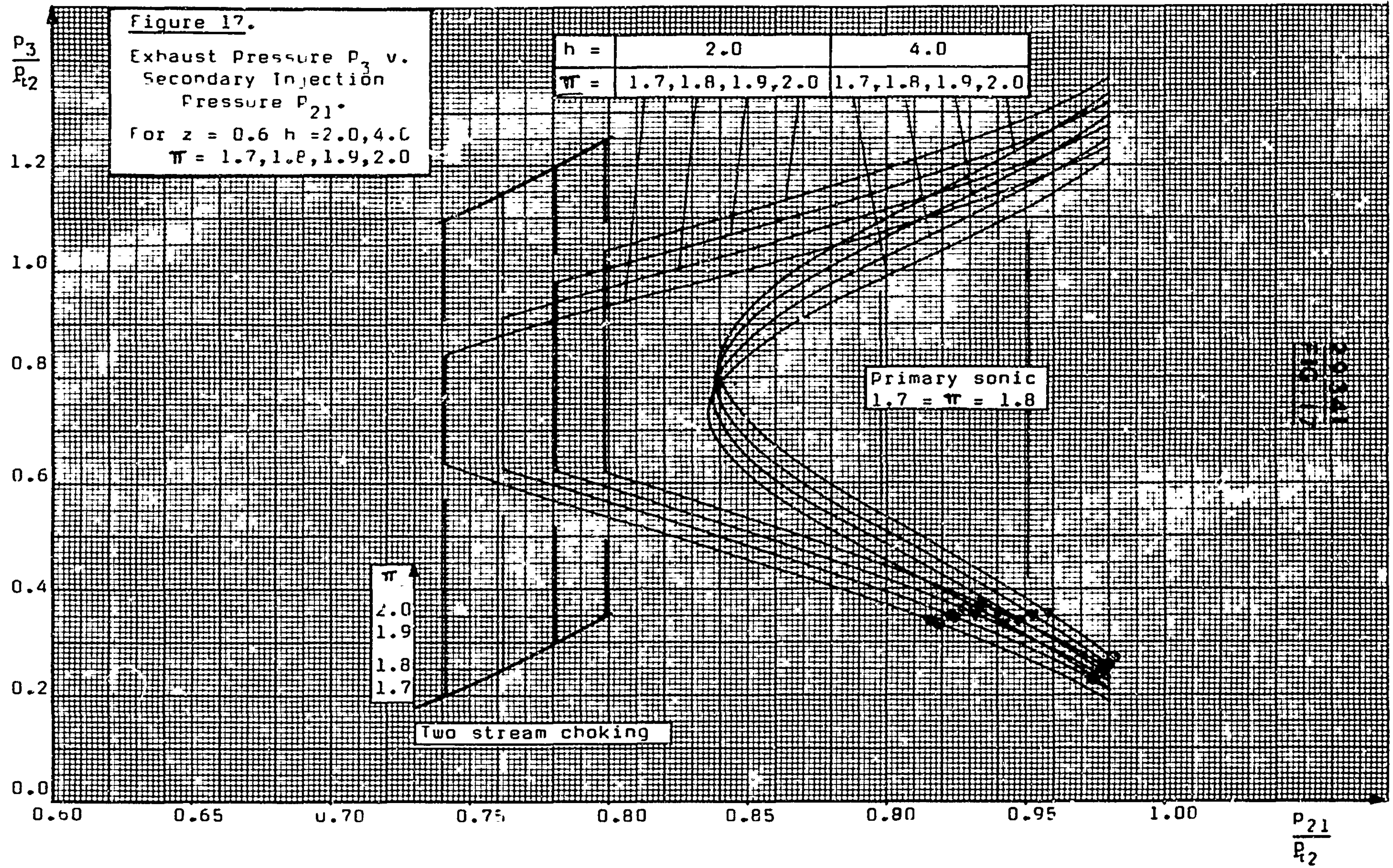
Full solutions for sonic injection



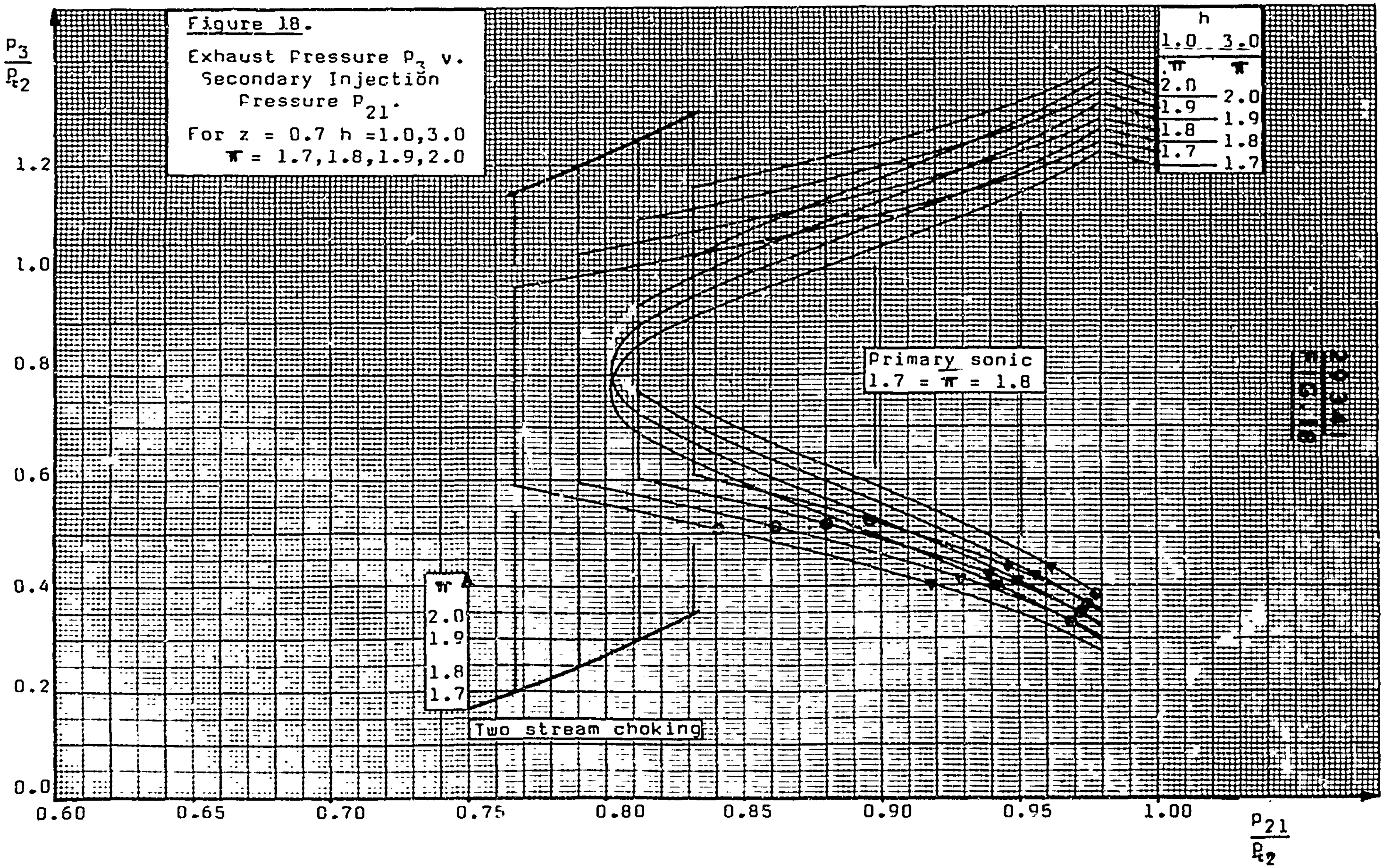
Full solutions for sonic injection



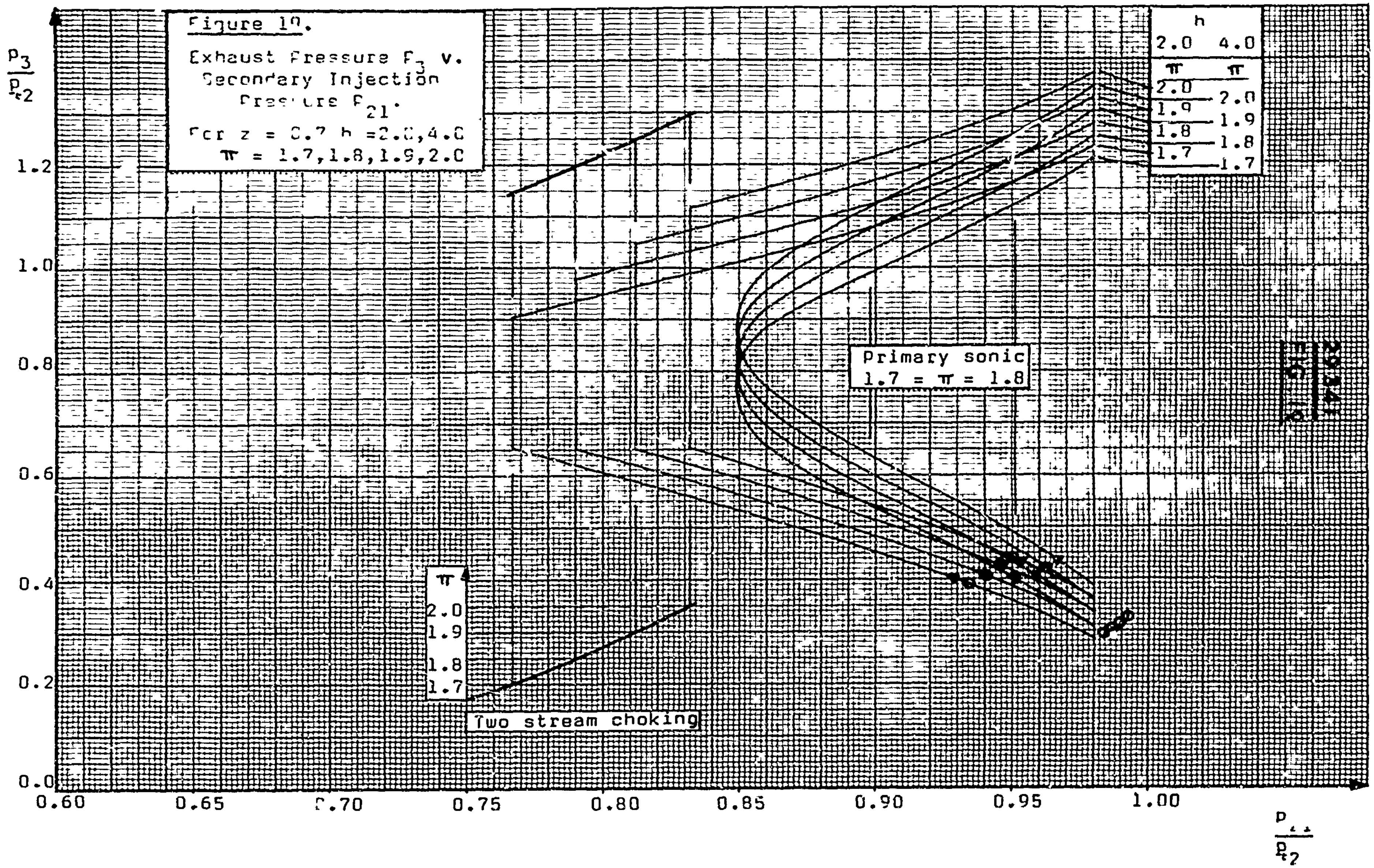
Full solutions for sonic injection



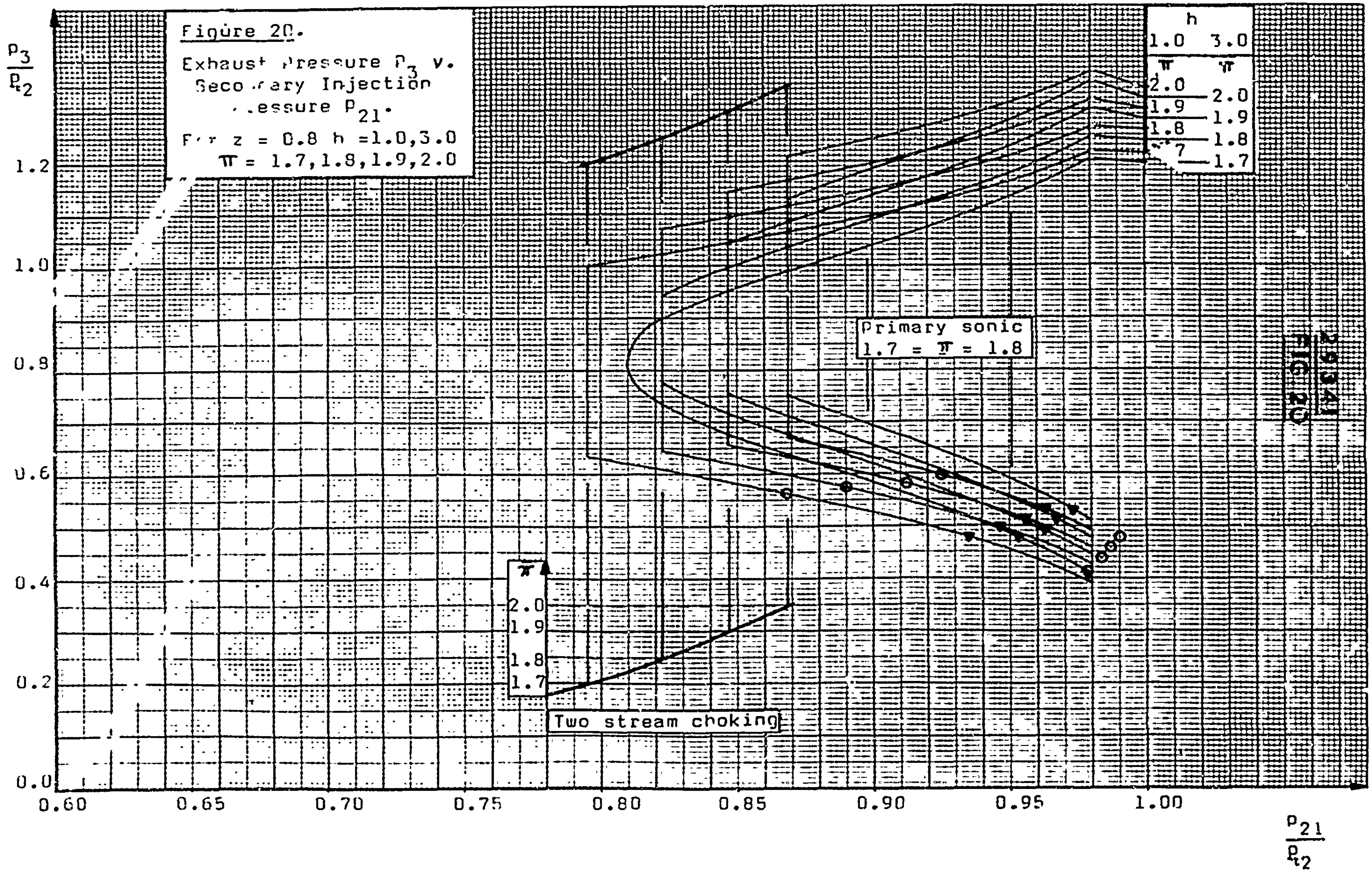
Full solutions for sonic injection



Full solutions for sonic injection



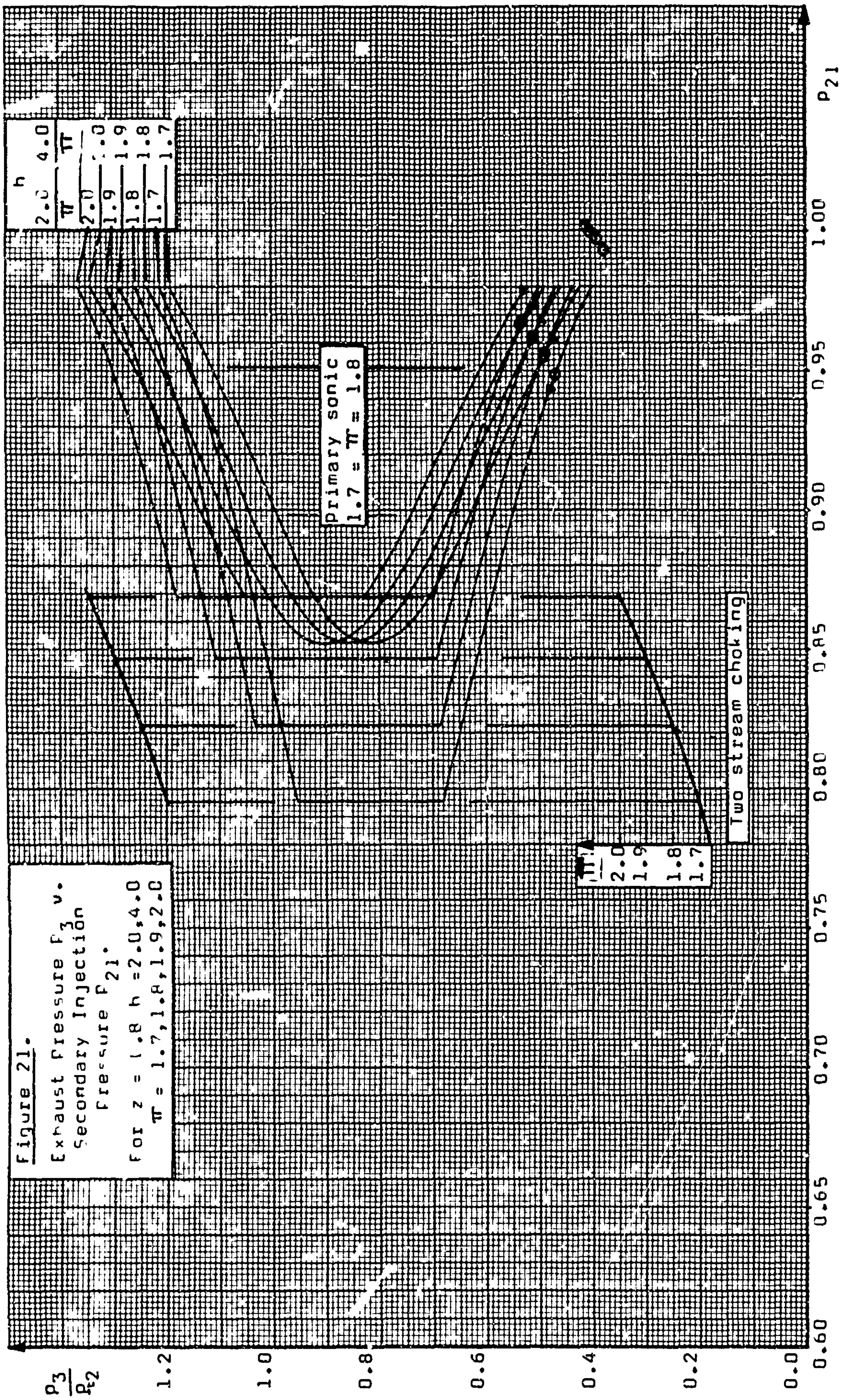
Full solutions for sonic injection



Full solutions for sonic injection

29 341

FIG. 21



Full solutions for sonic injection

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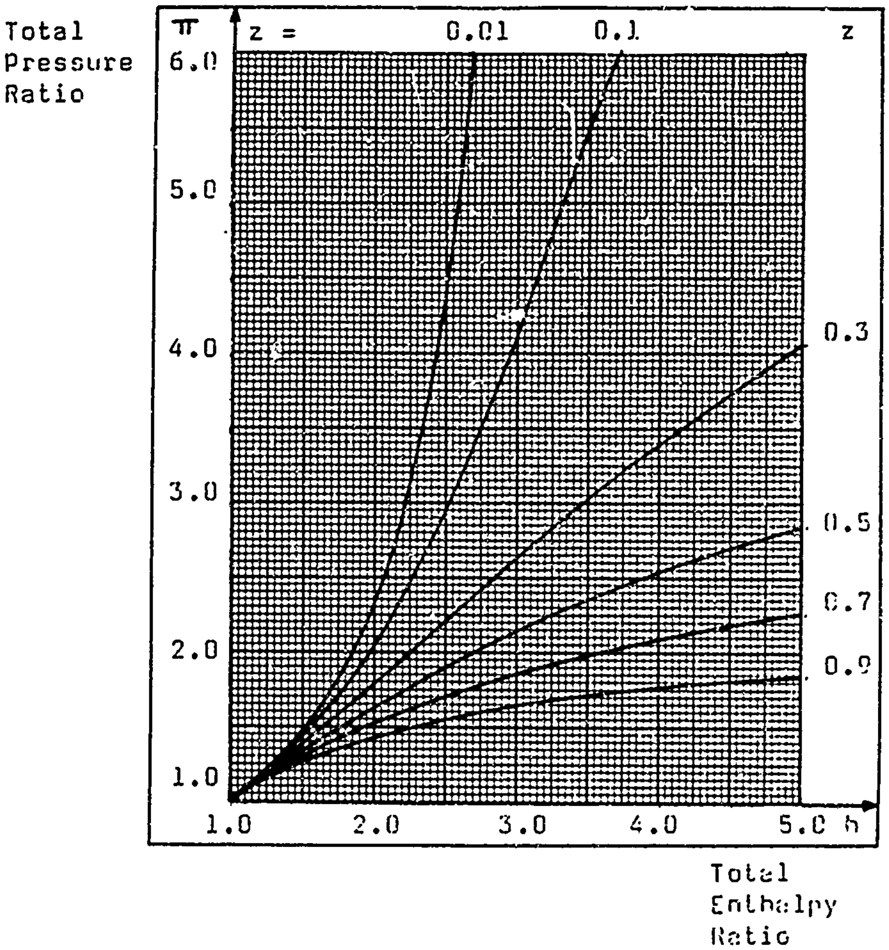
Figure 22.

Boundary between Choking Effects in the Supersonic Regime.



The domains of the three dimensional control parameter field, π - z - h, within which the system chokes due to the choking of the mixed stream and due to the two stream choking effect are divided by a surface containing those solutions that choke due to both effects simultaneously. This surface is shown in the figure above plotted as lines of constant z in the π - h field.

Boundary between choking effects in the supersonic regime.

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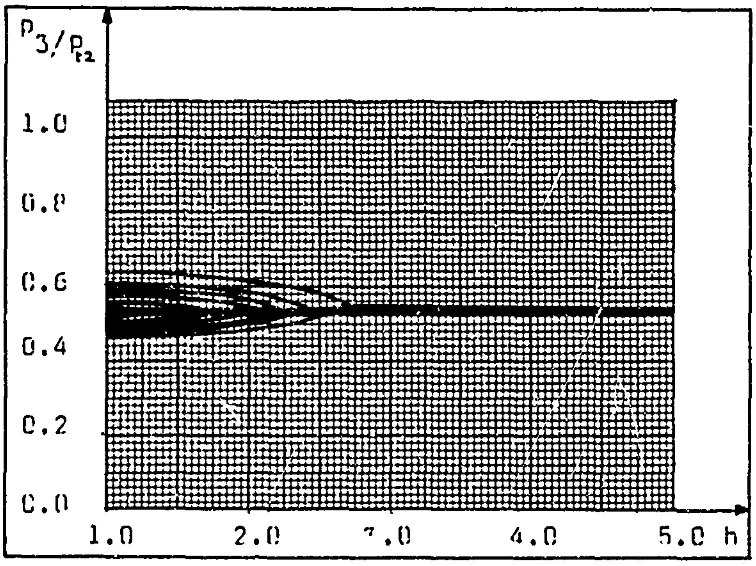
Figures 23 to 28.

In these figures the Supersonic Regime solutions are plotted as surfaces in the three dimensional π - h - P_3 field. Each figure represents a particular geometry or value of z. The solutions double valued in P_3 are those corresponding to two stream choking. The upper branches of these solutions are subsonic solutions and the lower branches supersonic. The surfaces are represented by lines of constant total pressure ratio, π, in the h - P_3 field.

Figure 23.

Geometric ratio z = 0.01.
Total pressure ratios π, from the inside out, 1.1, 1.3, 1.7, 2.5, 3.0, 4.0, 6.0.

Exhaust
Pressure

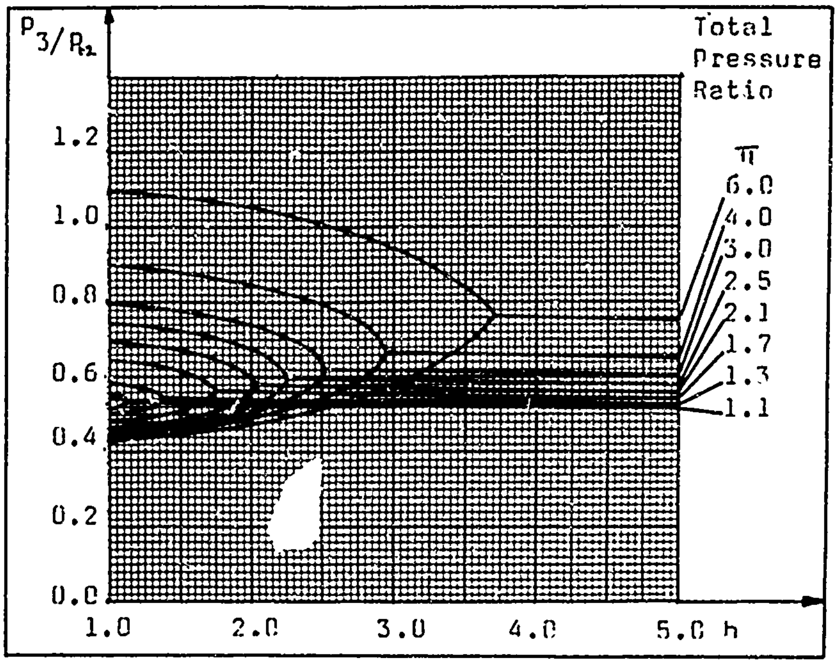


Total
Enthalpy
Ratio.

Solutions in the supersonic regime for sonic injection.

Figure 24. Geometric ratio z = 0.1

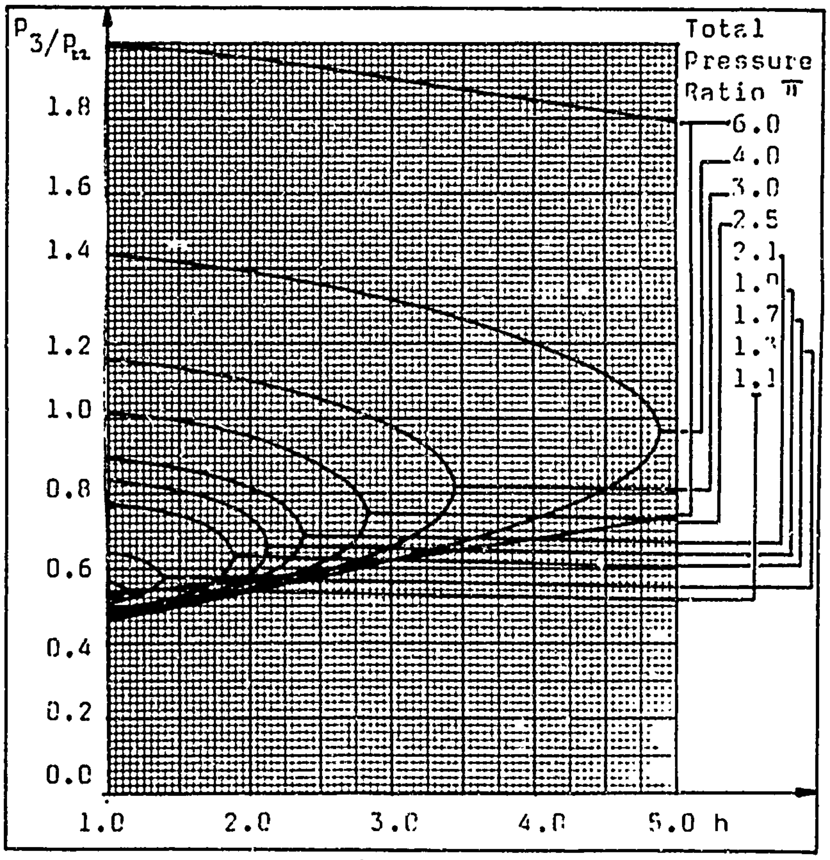
Exhaust Pressure



Total Enthalpy Ratio

Figure 25. Geometric ratio z = 0.3

Exhaust Pressure

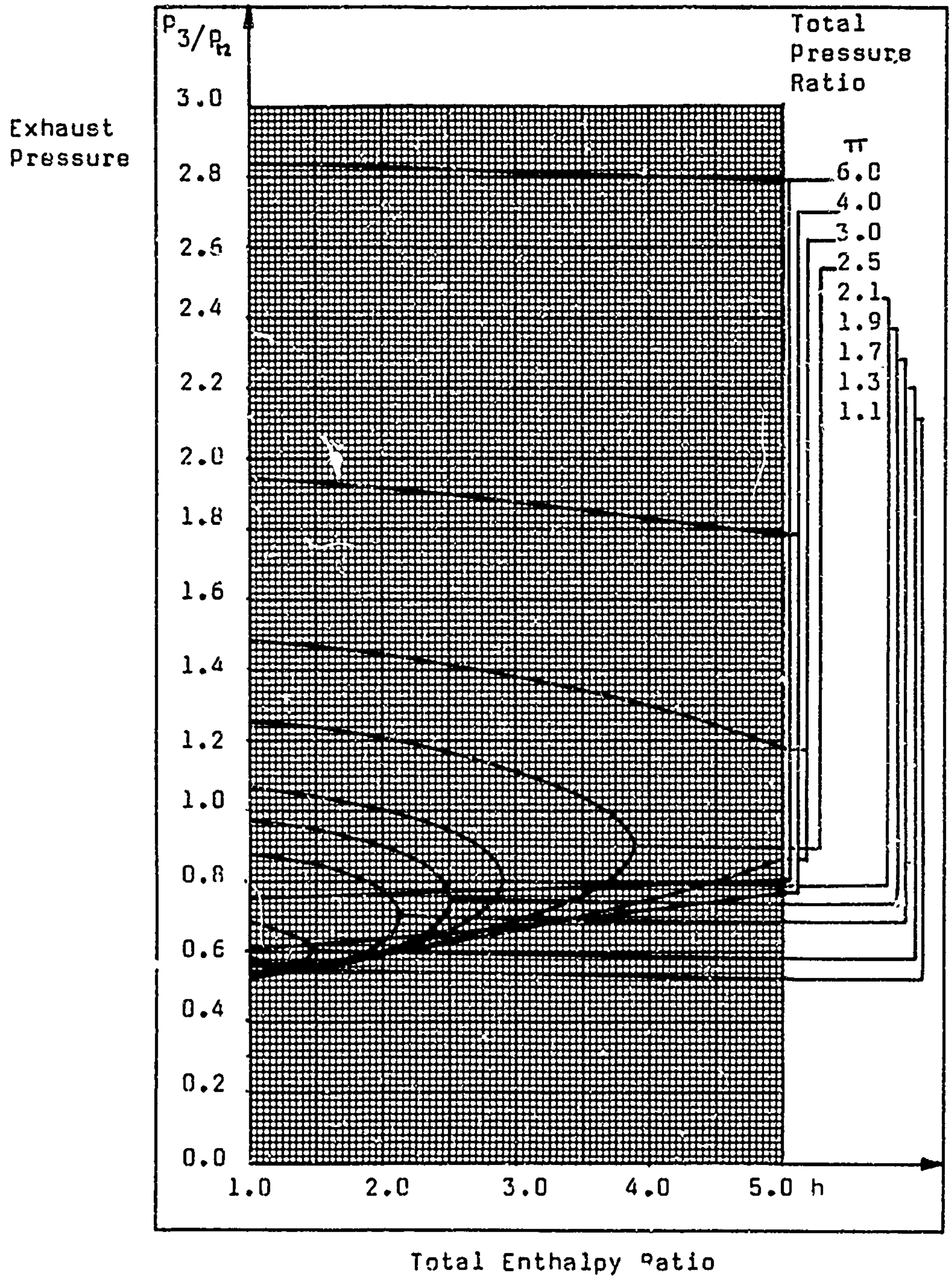


Total Enthalpy Ratio

Solutions in the supersonic regime for sonic injection.

Figure 26.

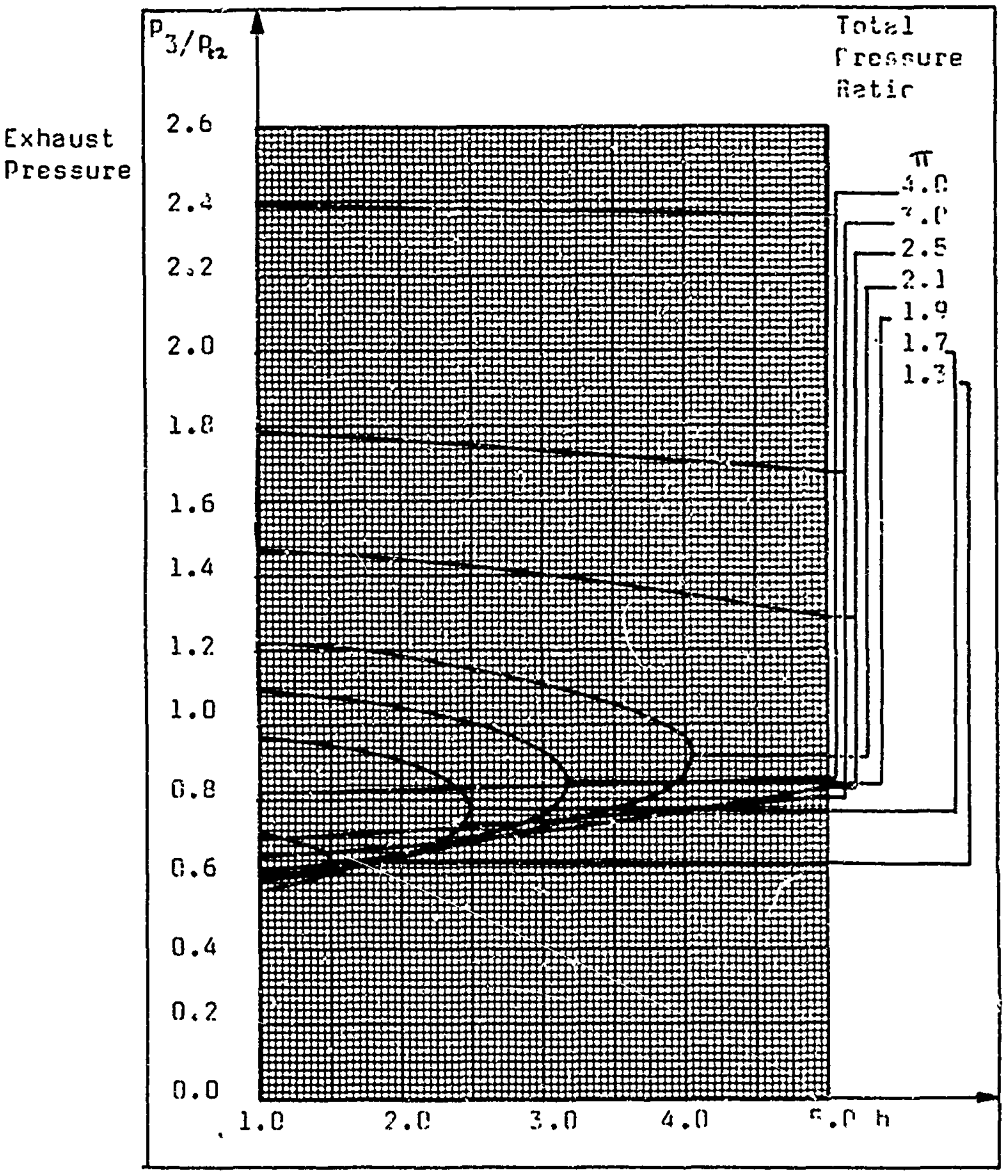
Geometric Ratio z = 0.5



Solutions in the supersonic regime for sonic injection.

Figure 27. **29341**

Geometric ratio z = 0.7



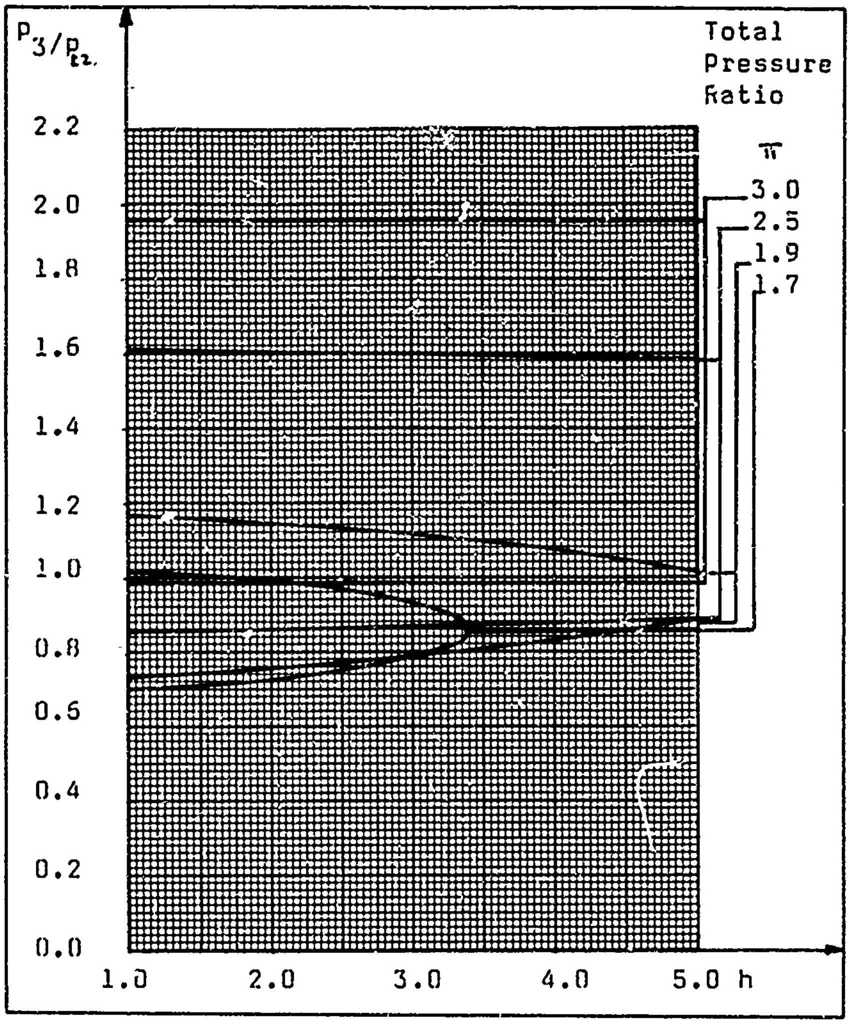
Total Enthalpy Ratio

Solutions in the supersonic regime for sonic injection.

Figure 28.

Geometric ratio z = 0.9

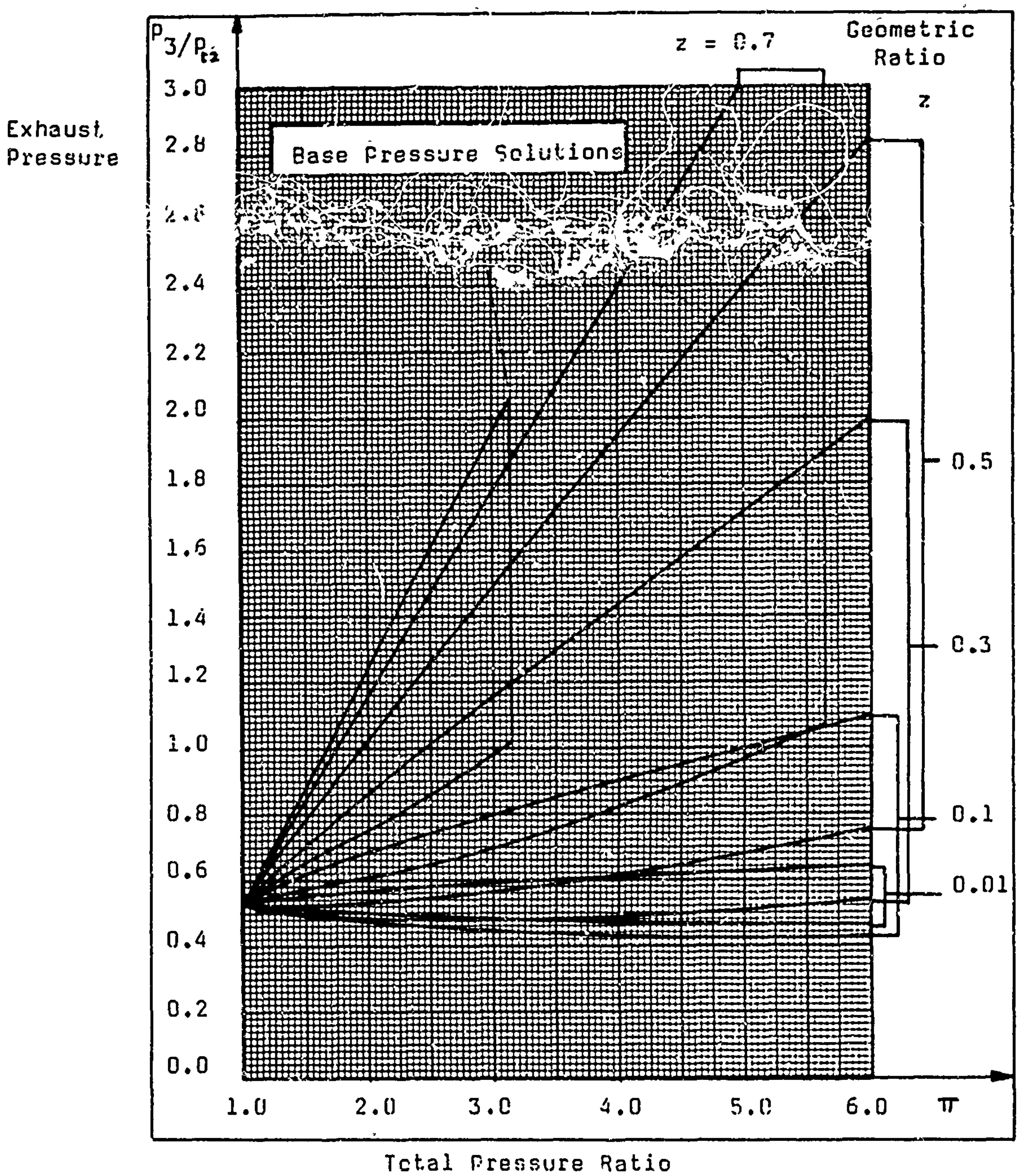
Exhaust Pressure



Total Enthalpy Ratio

Solutions in the supersonic regime for sonic injection.

Figure 29. Choked Solutions for Total Enthalpy Ratio of Unity.

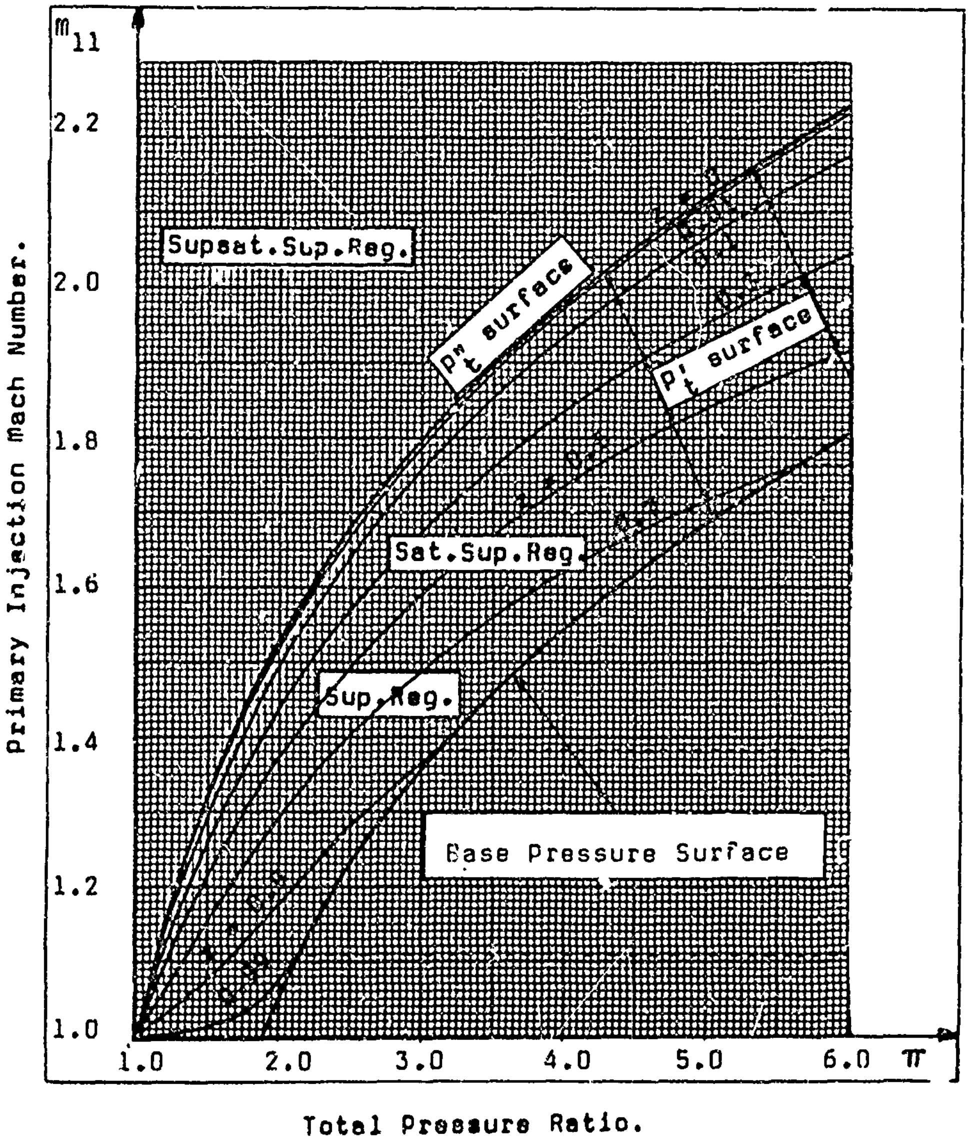


The maximum flow solutions, for the system in which the total enthalpies of the two streams are the same, for two surfaces in the three dimensional z-π-P_3 field. In the figure above these surfaces are shown as lines constant z in the π-P_3 field. The upper surface contains the subsonic solutions and the lower the supersonic solutions. The mixed flow will only choke in the special case when π = 1. For high values of z and π the solution enters the Base Pressure Regime. This is shewn by truncating the curves at the appropriate value of π.

Choked solutions for total enthalpy ratio of unity.

29341 FIG. 30

Figure 30 is a plan view of the three dimensional space formed by the control parameters M_{11}, π and z, projected onto the plane $z = 0$. The z axis thus may be considered to be normal to the plane of the paper. Three surfaces are depicted; two of these, the P_t'' surface and the Base Pressure surface, are independent of z and thus may be generated by lines normal to the paper. Their projections on the plane $z = 0$ are therefore single curves. The P_t' surface, however, is represented by a series of curves forming constant z contours. This surface lies between the other two, intersecting with the P_t'' surface along the common curve $z = 0$. Each point in the space corresponds to a particular ejector system operating at a specific total pressure ratio. The surfaces depicted divide the space into four separate volumes each representing a specific flow regime. The mode in which any particular ejector system will choke may be determined by finding in which volume the corresponding point lies. The volume between the plane $\pi = 1$ and the P_t'' surface represents the Supersaturated Supersonic Regime. The volume between the plane $M_{11} = 1$ and the Base Pressure surface contains the Base Pressure solutions. The volume between the P_t'' surface and the Base Pressure surface is divided into two by the P_t' surface; the part below the P_t' surface corresponds to the Supersonic Regime and the part above to the Saturated Supersonic Regime.



Boundaries of the supersonic regime for supersonic injection.